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Reputation and performance fee effects on portfolio choice by investment advisers¹

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Abstract

This paper examines a two-period model of investment management. Investors reallocate their wealth between two mutual funds managed by different investment advisers after observing the performance of each adviser in the first period. A reputation effect causes one adviser to choose a portfolio in the first period that is extreme given his private information about asset returns. Extreme portfolios are costly for risk-averse advisers and investors because mutual funds are riskier than in one-period or single-adviser settings. Adoption of a performance fee mitigates undesirable reputation effects and results in superior ex ante payoffs to investors. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

Investors who own mutual fund shares delegate their investment decisions to the advisers who manage these funds. Because investors can easily compare

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advisers' past performance and may switch advisers at any time, advisers face enormous pressure to produce a history of returns that compares favorably with those of their peers. The portfolio choice that maximizes investor perceptions of ability may differ from the choice that optimally trades off risk and return. Thus, reputation concerns can adversely affect welfare. This paper models these effects and shows how imposing a performance fee ameliorates the distortions caused by reputation concerns and improves welfare.

The question of whether mutual funds earn superior risk-adjusted returns has been the subject of extensive empirical investigation. In some circles, it is a puzzle why investors invest in actively managed mutual funds at all given that the typical mutual fund underperforms a risk-adjusted passive benchmark. A key assumption in this paper is that some adviser has private information that allows him to earn above-market risk-adjusted returns. Thus, investors' willingness to invest in mutual funds is rationalized by a belief that some managers have access to private information that enables them to outperform the market, even if these managers are mixed in a pool with uninformed charlatans who on average earn risk-adjusted returns that are at or below market returns. Because the informed adviser's information is not perfect, he might not outperform the market in every period. Because he receives information in each period, his performance is persistent. Performance persistence is consistent with many empirical studies.²

If some advisers are informed and others are not, investors should reallocate their wealth to funds with good track records. Consistent with the hypothesis that investors' wealth allocation decisions are based on inferences about advisers' abilities drawn from the history of mutual fund returns, portfolio composition and cumulative returns for funds are publicly reported and scrutinized by investors and rating agencies. Evidence in Brown et al. (1996), Falkenstein (1996), and Chevalier and Ellison (1997), is consistent with assets flowing to advisers with good track records and advisers altering the riskiness of the portfolio they manage to affect the flow of money into the fund they manage. Relatedly, Brown and Goetzmann (1995) show that poor performance is the strongest predictor of fund attrition.

Another well-established empirical phenomenon is that mutual fund advisers trade a great deal – mean dollar weighted turnover at US mutual funds is 73.8% annually (Sharpe, 1997). I show reputation concerns rationalize trading by uninformed managers.

² Grinblatt and Titman (1992), Lakonishok et al. (1992), Goetzmann and Ibbotson (1994), Hendricks et al. (1993) and Elton et al. (1996) document performance persistence in investment management. Brown et al. (1992) survey earlier evidence and consider the impact of survivorship bias. Grinblatt and Titman (1989b) find evidence that risk-adjusted returns of some growth funds were significantly abnormally positive over the years 1975 to 1984.

The compensation of investment advisers consists almost entirely of fees that are a percentage of net assets under management (an ‘asset fee’), and fees that depend on the realized performance of the adviser’s portfolio relative to a benchmark portfolio (a ‘performance fee’). Given this compensation arrangement, an adviser can increase his income in two ways: by choosing a portfolio that yields a high return, and by inducing investors to purchase additional units of the mutual fund. Many argue that garnering new funds is more important than earning high returns on invested funds, although the two objectives are related.³

This paper models the interaction among risk-averse investors and advisers in a two-period setting. It is common knowledge that one adviser is imperfectly informed about the future returns of a risky asset. Each period, each adviser receives fees from the investors whose assets he manages. At the end of the first period, investors reallocate their wealth based on their inferences about advisers’ abilities.⁴ Because asset fees are a fraction of assets under management, there is a benefit to advisers, realized in the second period, of appearing to be informed at the end of the first period. So, attempts to create a good reputation may distort first-period portfolio choices by advisers relative to the single-period optimum.⁵

I show how the reputation effect is influenced by the magnitude of the asset fee and the degree of risk aversion. Since investors correctly believe that some adviser is informed, assets flow to funds with good track records. When the barriers to entry into the mutual fund business are low, uninformed advisers enter and are willing to undertake risks to generate track records consistent with possession of superior information.

Imposing a performance fee changes advisers’ behavior so as to increase investors’ welfare. The key insight is that for every linear performance fee there exists a portfolio that is a mixture of the benchmark portfolio and the portfolio the adviser would choose in the absence of a performance fee that yields the same payoff as the adviser would earn in the absence of the performance fee. Thus, although the adviser can ‘undo’ any performance fee, imposing a

³ See, for example, Jonathan Clements, ‘Pilgrim Group, No Puritan, Irks More-Staid Funds’, *Wall Street Journal* (October 29, 1993) p. C1.

⁴ The opportunity to reallocate funds among advisers exists in settings other than the one described here. Many large institutions such as pension funds employ several advisers and exercise discretion over the amount allocated to each adviser. Some mutual funds have multiple advisers.

⁵ The offering documents for some mutual funds discuss the possibility that advisers’ actions may not conform to the fund’s objectives: ‘The Fund is subject to manager risk[.] The investment adviser manages the Fund according to the traditional methods of ‘active’ investment management, which involves the buying and selling of securities based upon economic, financial and market analysis and investment judgment. Manager risk refers to the possibility that the Fund’s investment advisers may fail to execute the Fund’s investment strategy effectively. As a result, the Fund may fail to achieve its stated objective’. (Vanguard Explorer Fund Prospectus, May 24, 1993, p. 5).

performance fee alters portfolio choice and hence a fee can be chosen to reduce the risk borne by mutual fund investors.⁶ This makes investors better-off. This result provides a new rationale for performance fees, namely: performance fees mitigate excessive risk-taking by informed and uninformed advisers alike.

Most previous work on the incentives of advisers assumes a one-period setting (e.g., Barry and Starks, 1984; Bhattacharya and Pfleiderer, 1985; Starks, 1987; Kihlstrom, 1988; and, Grinblatt and Titman, 1989a) and so does not address reputation. Research on multiperiod considerations in investment advisory relationships includes Heinkel and Stoughton (1994), and Huberman and Kandel (1993).

Heinkel and Stoughton consider an institutional client seeking investment advisory services from a pool of advisers. The authors derive characteristics of an optimal sequence of short-term contracts punctuated by renewal or firing decisions, and present a numerical example. As in this paper, advisers are of two types, implying that a similar signaling problem exists. In addition, if there is moral hazard in information gathering by advisers, then even though fully screening contracts are feasible, Heinkel and Stoughton show that offering a menu of contracts that causes most advisers to pool by selecting a 'boilerplate' contract is optimal, because this contract creates incentives for advisers to exert effort to gather information. For simplicity and tractability, I suppress moral hazard in information acquisition by advisers since, qualitatively speaking, the results on performance fees apply irrespective of the information structure, cost of effort, and endogenous effort choices that lead to an equilibrium. While Heinkel and Stoughton prove that a 'boilerplate contract' is optimal, I assume a single contract, analogous to their 'boilerplate contract', is offered to and accepted by all advisers. Consistent with Heinkel and Stoughton, advisers' relationships with investors are short term. In both settings, performance evaluation at the end of the first period is crucial to investors' and advisers' strategies and payoffs. In this paper, competition between advisers to develop a positive reputation leads to costly distortions in portfolio choices. These portfolio distortions cannot be treated within Heinkel and Stoughton's framework because they assume all investors and advisers are risk-neutral. Models with risk-neutral advisers are more tractable, but they (i) suppress potential salutary effects of risk aversion on adviser discipline and (ii) have limited implications for portfolio composition since expected returns drive portfolio choice irrespective of risk. Focusing on risk aversion raises issues that are complementary to but distinct from those considered in Heinkel and Stoughton.

⁶ Stoughton (1993) also derives a performance fee irrelevance result, though Stoughton's observation is different. He shows that advisers' incentives to gather costly information cannot be increased by imposing a performance fee.

Huberman and Kandel assume an exogenous benefit to reputation and focus only on the behavior of the adviser. They establish that both a separating equilibrium and a continuum of pooling equilibria can exist when the adviser is risk-averse. This paper extends their work in four ways. First, the costs and benefits of contracting arrangements that closely resemble actual practice are analyzed. Changes to the compensation arrangement induce changes in the expected value *and* risk of the fees paid under the arrangement. Both expected value and risk are shown to be useful in disciplining portfolio choices of risk-averse advisers. Second, explicitly modeling risk-averse investors and the flow of funds between advisers makes the reward to reputation endogenous in this paper and permits welfare comparisons across alternative structures. Since there are multiple advisers rather than just one, investors' inferences about advisers are conditioned on portfolio choice (as in earlier research) *and* performance relative to a peer group. Third, this paper shows how risk-aversion, fees, and the precision of private information interact to produce either a pooling or separating equilibrium. In contrast, earlier work does not characterize situations where particular equilibria emerge. Fourth, the stringent informational requirement in prior work that investors observe advisers' private signals is dropped.

Section 2 presents the model. That section identifies conditions under which separating and pooling equilibria occur when only asset fees are charged. Section 3 considers the effects of performance fees. Section 4 concludes the paper.

2. Model

The modeling objective is to illustrate how the desire to signal superior ability or obscure lack of ability can color investment decisions when advisers are endowed with different, privately known, but unobservable information. Initial assumptions are as follows.

2.1. Assumptions

(A1) *Mutual funds are price takers.* In the rational expectations literature (e.g., Admati and Pfleiderer, 1990), prices partly reveal information when trades based on private information are executed. The assumption here is that the funds run by advisers are price takers regardless of the trade they propose to undertake.

(A2) *Advisers move simultaneously.* In contrast to work by Scharfstein and Stein (1990) on herding and Gul and Lundholm (1995) on clustering in financial markets, the advisers in this game move simultaneously: advisers' strategies are

not conditioned on the actions of advisers who have already acted, or on the knowledge that to this point no one has acted.

(A3) *There is no collusion.* Fund advisers do not coordinate their investment decisions, make side payments, or pool their information.

(A4) *An adviser cannot make personal investments outside the fund he manages.* Grinblatt and Titman (1989a) explore performance fees in a one-period setting when the adviser can hedge the fee he receives under the management contract with the return on his personal portfolio. Here, the payoff to the adviser consists solely of the fees he receives under the management contract.

(A5) *One adviser is strictly better informed.* Employing more than one adviser may be desirable if advisers have different pieces of information. This possibility does not arise here because only one adviser has information.

(A6) *Fees are exogenous.* Fund advisers levy periodic fees that are a constant fraction of the assets under management. For purposes of this analysis, I treat the contractual form as an institutional fact in order to analyze the behavior of advisers. In practice, the variation in fees charged by advisers is small (Lakonishok et al., 1992). Many details of the contracts between investors and advisers are regulated by the Investment Advisers Act of 1940.

In Section 2.5, the last assumption is relaxed as I consider a larger game in which fees are endogenous and there is free entry by agents who become uninformed advisers.

2.2. Setup

2.2.1. Players, assets, and signals

There are two assets and many identical small investors.

The advisers and the investors exhibit the same preferences, described by the constant relative risk-aversion utility function for wealth $U(W) = W^\alpha/\alpha$ for some $\alpha < 1$. The investors and advisers have no wealth other than the amounts they will receive from the mutual fund at dissolution.

There are two assets, $\tilde{A}$ and M . Per dollar invested, risky asset $\tilde{A}$ returns 2 with probability $\frac{1}{2}$ and 0 with probability $\frac{1}{2}$ each period. The other asset, M , always returns the amount invested (i.e., the riskless return is normalized to zero). Interpret asset M as the market portfolio in an economy without systematic risk. Given these asset returns, a risk-averse uninformed investor would choose to invest all his wealth in the market portfolio, M , since it offers the same expected return as $\tilde{A}$.

Prior to investing, one adviser (the informed adviser) receives a private signal about the payoff to asset $\tilde{A}$. Half the time the adviser receives the signal G (good) and half the time he receives the signal B (bad). If he receives G , then the revised probability that $\tilde{A}$ will pay 2 is p where $\frac{1}{2} < p < 1$. Of course, with complementary probability $\tilde{A}$ will pay 0. If he receives B , then the revised probability that $\tilde{A}$ will pay 2 is $1 - p$. Uninformed advisers receive no signal.

The investors cannot invest in the assets directly. Instead, each investor must invest all her wealth in a single mutual fund run by one of the advisers.⁷ Suppose that half of the small investors invest with one adviser, and half with the other in period one.

2.2.2. One-period analysis

In a one-period or one-adviser setting, paying the adviser a fraction of the assets under management (computed at the end of the period) induces the adviser to make the portfolio choice that is optimal for investors conditional on the adviser's information.⁸ An informed adviser chooses the portfolio weight, ω^* , which maximizes:

$$EU(\omega\tilde{A} + (1 - \omega)M) = pU(1 + \omega) + (1 - p)U(1 - \omega). \quad (1)$$

The first order condition implies that the optimal weight is⁹

$$\omega^* = \frac{(H - 1)}{(H + 1)} \quad \text{where } H = \left(\frac{p}{1 - p} \right)^{1/(1 - \alpha)}. \quad (2)$$

An uninformed adviser invests his entire portfolio in asset M . As the precision of the signal increases, the optimal portfolio choice involves taking a more extreme position in the risky asset; however, for any value of $p < 1$, $\omega^* \in (-1, 1)$. This follows because the marginal utility for wealth becomes infinite as wealth approaches zero and a portfolio choice of $\omega = 1$ or $\omega = -1$ exposes the adviser

⁷ If investors could divide their wealth between the two mutual funds, they would do so. In the first period, the decision would be to allocate half of their wealth to each portfolio. In the second period, investors would reallocate their wealth between the two portfolios based on the realized return and investment decisions. If both advisers choose portfolios that signal private information, then investors cannot conclude for certain which adviser is informed. They do know that the adviser whose portfolio produced the best return is more likely to be informed. Accordingly, investors would place more (but not all) of their wealth with the adviser who had the better first-period return. Allowing investors this freedom makes it more difficult to derive advisers' first-period portfolio choices, though the same intuitions apply. Often, mutual funds require a minimum contribution of \$5000 or more. For many individual investors, this investment floor may preclude them from dividing their money among the funds with similar objectives over which comparisons of managers' abilities are drawn.

⁸ The size of the fraction can be chosen to satisfy the participation constraints of the investors and advisers. That is, paying the adviser a fraction of the ending value of the mutual fund is an optimal contract in a one-shot game because (i) signaling has no role; and, (ii) given the assumptions about utility, wealth, and the costless arrival of information, no agency problem or other conflict induces any difference in investor and adviser preferences with respect to portfolio choice. The optimality of this contract in the one-shot game highlights the distortions introduced by reputation effects in the two-stage game.

⁹ The utility function $U(W) = \log W$ corresponds to risk aversion parameter $\alpha = 0$. In this case, $\omega^* = 2p - 1$.

to the possibility of zero wealth in some state. Let $\hat{\omega}$ and $\check{\omega}$ be the solutions to Eq. (1) when the informed adviser receives signals G and B , respectively. By symmetry, $\hat{\omega} = -\check{\omega}$.

2.2.3. Two-period Analysis

Suppose there are just two advisers; investors know exactly one of the advisers is informed; the informed adviser receives a signal at the beginning of each period; signals and outcomes are serially independent; and, initially, the investors believe it is equally likely that either adviser is the informed one. At the end of the first period, each investor may stay put or switch her wealth from one fund to the other. The investors observe the composition of the portfolios chosen by the advisers. The performance and portfolio weights of the two funds help investors divine which adviser is informed. In particular, if the investors observe that an adviser chooses a portfolio weight of $\omega = 0$, then they conclude the adviser is uninformed. After the investors reallocate their wealth between advisers, the informed adviser again receives a signal. Both advisers then pick portfolios for period two. Consumption takes place after period two returns are realized. The precision of the signal received by the informed adviser, p , is assumed to be common knowledge. Table 1 summarizes these facts.

Table 1
Summary of model

There are two periods and two advisers. At the beginning of both periods, each adviser divides the funds he has under management between the risky asset and the riskless asset. One adviser receives an informative signal about the return on the risky asset; the other does not. The returns on the risky asset and both portfolios are realized and known to all parties. Based on the realized asset and fund returns at the end of the first period, investors choose to reallocate their wealth between the funds managed by the advisers. Advisers are concerned about the reputation they develop in the first period because they are paid according to the assets they have under management. If an adviser's reputation at the end of the first period is adverse, he may have no assets to manage in the second period. If his reputation is favorable he manages all investors' wealth in the second period. Both advisers and all investors seek to maximize their expected utility from consuming all their wealth at the end of period 2.

	Time			
	Period 1		Period 2	
Informed Adviser	Receives signal and invests	Return realized	Receives signal and invests	Return realized
Uninformed Adviser	Invests	Return realized	Invests	Return realized
Investors		Return realized	Reallocate investment between advisers	Return realized

Since the advisers receive a fee for managing the portfolio that is a fraction of the assets under management, they care about investors' perceptions of ability. As soon as the investors conclude that an adviser is uninformed, they cease to allocate any wealth to him, and in the succeeding period, the adviser's fee income is zero because the fund he manages has no investors. To forestall this possibility, the uninformed adviser has incentives to appear informed. Assuming the informed adviser chooses either portfolio weight $\hat{\omega}$ or $\check{\omega}$ (depending on whether he receives signal G or B), the uninformed adviser can mimic the informed adviser at least some of the time by choosing portfolio weight $\hat{\omega}$ with probability q and $\check{\omega}$ with probability $1 - q$. Table 2 lists the possible outcomes. Choices of q outside $[1 - p, p]$ reduce the likelihood the uninformed adviser manages some or all the assets in period 2 (see Lemma A.1 in the appendix for a proof). We suppress further consideration of these cases.

Since each investor is constrained to place all her wealth with one adviser or the other, then for any $q \in [1 - p, p]$ the best response for an investor is to place her wealth in the second period with the adviser who performs best in the first period. In the event of a tie, the investor is indifferent to whether she invests her money in period two with one adviser or the other. The effect of mimicry on the adviser's reputation is less severe than the effect of choosing $\omega = 0$. When an adviser picks $\omega = 0$, investors are certain the adviser is uninformed. When one adviser chooses $\hat{\omega}$ while the other chooses $\check{\omega}$, investors can only conclude that, with probability p , it is the uninformed adviser who performed worst.

Let ω_{it} be the fraction of the assets under management that adviser i places in asset $\tilde{A}$ in period t . Then one dollar invested in adviser i 's portfolio returns

$$\tilde{R}_{it} = \omega_{it}\tilde{A} + (1 - \omega_{it})M \quad \text{for } i \in \{I, U\},$$

where I and U denote the informed and uninformed advisers, respectively. Both advisers levy a management fee of f per dollar of assets invested with the adviser at the end of each period. Restrict f to lie in $(0, 1)$.¹⁰ Then in period t an investor receives either $(1 - f)\tilde{R}_{It}$ or $(1 - f)\tilde{R}_{Ut}$ per dollar invested. The investor's objective is to maximize her expected utility at the end of two periods assuming she remains fully invested in one of the mutual funds offered by adviser 1 and adviser 2.

Let γ_{it} be the proportion of total funds invested with adviser i at the beginning of period t . Fix $\gamma_{i1} = \frac{1}{2}$ for $i \in \{I, U\}$. The proportion γ_{i2} depends on the information available at the end of period one, namely the portfolio choices of the two advisers and the realized outcomes, as well as the strategies of the investors. If the advisers choose the same portfolio, then the investors' posterior beliefs about the abilities of the advisers are the same as their prior beliefs because the investors have received no information. An inference about the

¹⁰Typically, f is about 1% per year in an equity mutual fund.

Table 2
Summary of signals, actions, and outcomes in the pooling equilibrium

The signal seen by the informed adviser is either G or B . With probability p , asset A is worth 2 (0) following signal G (B). The informed adviser chooses portfolio weight $\hat{\omega}$ following G and $\tilde{\omega}$ following B . The uninformed adviser randomly chooses either $\hat{\omega}$ or $\tilde{\omega}$.

Case	1	2	3	4	5	6	7	8
Signal	G	G	G	G	B	B	B	B
Payoff on asset A	2	2	0	0	0	0	2	2
Informed adviser's portfolio choice	$\hat{\omega}$	$\hat{\omega}$	$\hat{\omega}$	$\hat{\omega}$	$\tilde{\omega}$	$\tilde{\omega}$	$\tilde{\omega}$	$\tilde{\omega}$
Uninformed investor's portfolio choice	$\hat{\omega}$	$\tilde{\omega}$	$\hat{\omega}$	$\tilde{\omega}$	$\hat{\omega}$	$\tilde{\omega}$	$\hat{\omega}$	$\tilde{\omega}$
Outcome:								
Informed adviser outperforms uninformed adviser tie	X	X	X		X	X		X
Uninformed adviser outperforms informed adviser				X			X	
Probability of outcome assuming the uninformed Adviser chooses portfolio weights $\hat{\omega}$ and $\tilde{\omega}$ with probabilities q and $1 - q$, respectively	$\frac{pq}{2}$	$\frac{p(1-q)}{2}$	$\frac{(1-p)q}{2}$	$\frac{(1-p)(1-q)}{2}$	$\frac{pq}{2}$	$\frac{p(1-q)}{2}$	$\frac{(1-p)q}{2}$	$\frac{(1-p)(1-q)}{2}$

identity of the informed adviser can only be drawn when the actions of the advisers differ.

Each adviser's objective is to maximize his expected utility at the end of two periods. Normalize aggregate initial wealth to unity since the problem is independent the level of wealth. Let

$$\tilde{I} = \gamma_{I1}\tilde{R}_{I1} + \gamma_{U1}\tilde{R}_{U1}$$

be the aggregate wealth of investors and advisers at the end of period one. In each period the fee earned by an adviser is the fee per dollar of assets, f , times the fraction of investors' wealth allocated to the adviser, γ_{it} , times investors' total wealth at the beginning of the period (1 in the first period and $(1 - f)\tilde{I}$ in the second period), times the return on those funds, $\tilde{R}_{it}$. For each adviser, expected utility is calculated with respect to the fee earned in the first period times the return earned on that fee income in the second period, $\tilde{R}_{i2}$, plus the fee earned in the second period. For adviser i , this is

$$EU(f\gamma_{i1}\tilde{R}_{i1}\tilde{R}_{i2} + f\gamma_{i2}(1 - f)\tilde{I}\tilde{R}_{i2}) = EU(f\tilde{R}_{i2}(\gamma_{i1}\tilde{R}_{i1} + \gamma_{i2}(1 - f)\tilde{I})). \quad (3)$$

In this formulation, the adviser has utility for wealth at the end of two periods as there is no intermediate consumption.¹¹

2.3. Characterizing equilibria

We solve for Bayes Nash, subgame perfect equilibria by backward induction.

2.3.1. Optimal investment policy in period two

In period two, there is no benefit to the adviser from improving the investors' assessments of his ability. With a power utility function, the portfolio weights chosen by the adviser are independent of (i.e., multiplicatively separable from) the adviser's wealth and the size of the fee.¹² Also, the adviser chooses to invest both the fee he earned in the first period and money he manages on behalf of the investor identically. Moreover, since the investor is assumed to have preferences identical to those of the adviser, the portfolio selected by the adviser is the one that would be chosen by the investor if she were endowed with the same information as the adviser. Thus, as shown in Section 1.2, in the second period,

¹¹ Although each adviser manages some investor wealth in the the first period, neither adviser has any personal wealth until management fees are paid at the end of the period. The adviser must invest the fee he receives at the end of the first period in the fund he manages. An adviser cannot invest his own wealth (or the wealth of his clients) in the fund managed by the other adviser.

¹² For $U(W) = \log W$, portfolio weights chosen by the adviser are additively separable from the adviser's wealth.

the uninformed adviser chooses portfolio weight $\omega = 0$. The informed adviser chooses $\hat{\omega}$ on receipt of signal G and $\check{\omega}$ on receipt of signal B .

2.3.2. Optimal investment policy in period one

This section introduces the notation used to describe equilibria, makes some preliminary observations about the nature of equilibria, and provides an overview of the pooling and separating equilibria that are considered in more detail later. A strategy for the uninformed adviser is $s_U = (\Omega; \Pi)$ where $\Omega = (\omega_1, \dots, \omega_n)$ is a vector of first period portfolio choices and $\Pi = (\pi_1, \dots, \pi_n)$ is a vector of probabilities. The probability of playing ω_i is π_i and $\sum_i \pi_i = 1$. A strategy for the informed adviser is $s_I = (\Omega; \Pi^B, \Pi^G)$ where Ω is a vector of portfolios as above and Π^j is a vector of probabilities. The probability of playing ω_i after receiving signal j is π_i^j and $\sum_i \pi_i^j = 1$ for $j \in \{G, B\}$.

A fund investor's pure strategies are $s_F(\omega^1, \omega^2; R_{11}, R_{21}) \rightarrow \{\text{stay put, switch}\}$ where ω^i denotes the portfolio choice of adviser i .¹³ Each investor decides whether to stay invested in the mutual fund she held in the first period or switch to the other fund on observing the portfolio choices and returns of the two advisers. Assume investors choose to stay put in the event both advisers choose the same portfolio. When advisers' portfolio choices differ, all investors invest with the adviser they infer is most likely to be informed. Then, the fraction of aggregate investor wealth managed by adviser i in the second period can take one of only three values. That is, $\gamma_{i2} \in \{0, \frac{1}{2}, 1\}$.

There are many sequential equilibria to this game because there are no restrictions on the beliefs of investors when they observe a portfolio choice by an adviser that is off the equilibrium path of the game. Portfolio choices for the informed and uninformed advisers can be supported in equilibrium by supposing investors rationally conclude that an adviser who deviates from the equilibrium play is surely uninformed. An adviser who investors believe is uninformed receives no assets to manage in the second period. Such beliefs can be used to enforce a wide range of equilibria in which advisers play mixed strategies.

Nevertheless, it is possible to provide some characterization of equilibrium play. Suppose the uninformed adviser chooses $\omega \neq 0$ with positive probability and the informed adviser never plays ω in some equilibrium. Then the investors conclude that any adviser who chooses ω is uninformed, so an adviser who plays ω is allocated no funds by the investors in period two. That is, the period two outcome is the worst one possible when an adviser plays ω . Also, ω is strictly dominated by $\omega_0 = 0$ for the uninformed adviser in period one. Since ω is

¹³ Note that here, $i \in \{1, 2\}$ rather than $i \in \{I, U\}$, because investors do not know which adviser is informed.

strictly dominated by ω_0 , ω cannot be chosen by the uninformed adviser in equilibrium.

Remark 1. If the uninformed adviser chooses portfolio $\omega \neq 0$ with positive probability in some equilibrium, then the informed adviser must choose ω with positive probability as well.

Thus, any equilibrium in which the uninformed adviser chooses $\omega \neq 0$ must involve some pooling. Note also that when the uninformed adviser chooses $\omega \neq 0$, investors bear avoidable risk and are not compensated by superior expected returns. This is a cost borne by investors due to reputation concerns on the part of the uninformed adviser.

Reputation concerns can also distort the investment choices of the informed adviser, who may separate from the uninformed adviser via his portfolio choice. To illustrate and contrast these two effects, the remainder of this paper treats only two of the sequential equilibria to this game. These equilibria are simple and appealing because the informed adviser plays a pure strategy. They serve to illustrate the fundamental tensions that all equilibria share. We introduce them here and prove their existence in the next two subsections. The first one is a pooling equilibrium,

$$\{s_U((- \omega, \omega); (q, 1 - q)), s_I((- \omega, \omega); (1, 0), (0, 1))\},$$

where $q \in (1 - p, p)$ and ω is close to $\hat{\omega}$, the optimal one-period portfolio choice of the informed adviser conditional on signal G . The second is a separating equilibrium,

$$\{s_U(0; 1), s_I((\check{\omega}, \hat{\omega}); (1, 0), (0, 1))\}.$$

In the second equilibrium, $\check{\omega}$ and $\hat{\omega}$ are portfolio choices that are more extreme (i.e., further from zero) than $\check{\omega}$ and $\hat{\omega}$.

2.3.3. Pooling equilibrium

Suppose the informed adviser chooses the portfolio that is optimal (in a one-period setting ignoring reputation effects) given his information (i.e., $s_I((\hat{\omega}, \check{\omega}); (1, 0), (0, 1))$). In the pooling equilibrium, the uninformed adviser mimics the informed adviser by randomly choosing either $\hat{\omega}$ or $\check{\omega}$. Two conditions need to be satisfied for the pooling equilibrium to exist. First, the pooling equilibrium will only exist when the contribution to expected utility associated with the lottery in which the uninformed adviser gets either some, all, or none of the investors' funds to manage in the second period more than offsets the loss in expected utility from making a risky portfolio choice in the first period. Second, there must not exist a deviation from the pooling equilibrium that investors could attribute only to the informed adviser and that is attractive to the informed adviser relative to equilibrium outcomes even though investors give

the deviator no funds to manage in the second period.¹⁴ The first proposition establishes conditions under which the only equilibria are pooling equilibria.

Proposition 1. Suppose $\alpha > \frac{1}{2}$ and

$$f < 2 - 2^{(1-\alpha)/\alpha}. \quad (4)$$

Then (i) no separating equilibrium exists, and (ii) the following pooling equilibria exist

$$s_U((-\omega, \omega); (q, 1 - q)),$$

$$s_I((-\omega, \omega); (1, 0), (0, 1)),$$

where $q \in (1 - p, p)$ and $\omega \in (\hat{\omega} - \delta, \hat{\omega} + \delta)$ for some $\delta > 0$. Furthermore, these pooling equilibria survive the Intuitive Criterion.

All proofs are in the appendix. The Intuitive Criterion is described in Cho and Kreps (1987). The Intuitive Criterion is an equilibrium refinement that rules out certain sequential equilibria in signaling games on the grounds that such equilibria are supported by unreasonable beliefs off the equilibrium path. In our game, imposing the Intuitive Criterion restricts investors' beliefs in the event the portfolio choice of one of the advisers is off the equilibrium path in the following two ways: First, if the payoff to the informed adviser in equilibrium exceeds the best (over all possible reallocation choices by investors) payoff from a deviation – clearly, this is allocating all funds to the deviator in the second round – then investors' beliefs must assign zero probability to the possibility the deviator is informed. Similarly, if the payoff to the uninformed adviser in equilibrium exceeds the best (over all possible reallocation choices by investors) payoff from a deviation – again, this is allocating all funds to the deviator in the second period – then investors' beliefs must assign zero probability to the possibility the deviator is informed. Imposing the Intuitive Criterion in Proposition 1 does not reduce the set of sequential equilibria because, for an adviser of either type, the equilibrium payoff is less than the best payoff from a deviation, namely managing all investors' funds for sure in the second period, so no off-equilibrium beliefs are ruled out. In particular, investor beliefs that any deviator is uninformed, which support the equilibrium in Proposition 1, are not ruled out by the Intuitive Criterion.¹⁵ However, when we consider separating equilibria in Proposition 2, the Intuitive Criterion does eliminate some unreasonable beliefs

¹⁴ Such a deviation gives rise to the separating equilibrium described in the next section.

¹⁵ Likewise, restricting consideration to Pareto-optimal equilibria does not narrow the set of equilibria because, in general, the derivatives of the objective functions of uninformed and informed advisers, and investors may differ in sign at $\hat{\omega}$.

and equilibria. For comparability, we state the effect of applying the Intuitive Criterion in Proposition 1 as well.

For any $p \in (\frac{1}{2}, 1)$, condition (4) describes a convex region in (α, f) -space. As α approaches 1, investors and advisers become risk neutral. At this point, inequality (4) holds for any asset fee $f < 1$. Separation is impossible when advisers are risk neutral because the uninformed adviser assesses the same one-period expected return on every portfolio. Since mimicry offers higher expected returns in the second period, the informed adviser will mimic. As α decreases, the informed adviser becomes more risk averse, so mimicking the informed adviser becomes more costly while the second period benefits of mimicry are unchanged. Thus, it is intuitive that the right-hand side of Eq. (4) is decreasing in α .

The asset fee, f , also affects the nature of the equilibrium. As f increases, more of the initial wealth of investors is owned by advisers at the end of the first period. Consequently, the second-period fee income of advisers, expressed as a fraction of total income over both periods, decreases. Hence, as f grows larger, the second-period fee income of the uninformed adviser becomes less important to his overall utility (i.e., the relative value of reputation diminishes). Since the first-period contribution to the overall utility of the uninformed adviser is maximal when portfolio $\omega_0 = 0$ is chosen, a high asset fee facilitates separation. Inequality (4) is harder to satisfy as f increases. For small values of f , inequality (4) holds for a wide range of α . For example, for $f = 1\%$, inequality (4) holds for $\alpha \in (0.5018, 1)$; for $f = 3\%$, inequality (4) holds for $\alpha \in (0.5055, 1)$.

2.3.4. Separating equilibrium

Suppose the uninformed adviser prefers mimicking the informed adviser to choosing the market portfolio $\omega_0 = 0$ when the informed adviser follows strategy $s_I((\hat{\omega}, \check{\omega}); (1, 0), (0, 1))$. To discourage mimicry, the informed adviser could use his information more aggressively. That is, the adviser places a more extreme weight on asset $\tilde{A}$ than he would choose in the absence of reputation concerns.

If his information suggests that $\tilde{A}$ is more likely to pay 2 than 0, then the informed adviser may choose $\hat{\omega} > \check{\omega}$, thereby over-investing in $\tilde{A}$. If the information suggests that $\tilde{A}$ will have lower returns in the coming period, then the adviser will choose $\check{\omega} < \hat{\omega}$, thereby over-investing in M . The informed adviser will choose $\hat{\omega}$ and $\check{\omega}$ at levels that just discourage the uninformed adviser from mimicking him. The portfolio choices of $\hat{\omega}$ following G and $\check{\omega}$ following B impose a cost on the informed adviser in the first period relative to $\hat{\omega}$ and $\check{\omega}$, but this cost is offset in the second period if it discourages the uninformed adviser from mimicking. Then in period two, the informed adviser manages all investors' wealth for sure. In the second period, the investors invest with the adviser who chooses $\hat{\omega}$ or $\check{\omega}$. Under the separating equilibrium, an adviser whose portfolio performs badly but whose actions indicate that he had private

information manages all wealth in period two. If both advisers or neither adviser plays $\hat{\omega}$ (an event off the equilibrium path), the investors stay put. The next proposition establishes the existence of a separating equilibrium for cases where advisers are more risk averse than in Proposition 1.

Proposition 2. Suppose $\alpha \leq -1$. The unique sequential equilibrium satisfying the Intuitive Criterion is

$$s_U = (\omega_0; 1) \quad \text{and} \quad s_I = (\hat{\omega}, \check{\omega}; (1, 0), (0, 1))$$

where $\check{\omega} = -\hat{\omega}$, $\hat{\omega} = \max(\hat{\omega}, \bar{\omega})$, $\bar{\omega}$ is the unique solution to

$$\begin{aligned} U(1-k) = & \frac{p}{2}U(1+\omega) + \frac{1-p}{2}U(1-\omega) + \frac{p}{2}U((1-\omega) - k(1-\omega)) \\ & + \frac{1-p}{2}U((1+\omega) + k(1-\omega)), \end{aligned} \quad (5)$$

and $k = (1-f)/(2-f)$.

Choosing portfolio weights $\hat{\omega} > \hat{\omega}$ and $\check{\omega} < \check{\omega}$ is strictly less costly in the first period for the informed adviser than for the uninformed adviser. To see this, consider a small increase, ε , in the portfolio weight on asset $\tilde{A}$ following signal G . Since $\hat{\omega}$ is the optimal one-period weight for the informed adviser, the change in the expected utility of the informed adviser in moving from $\hat{\omega}$ to $\hat{\omega} + \varepsilon$ is approximately zero. For the uninformed adviser, the slope on first-period expected utility at $\hat{\omega}$ is strictly negative. Moreover, the slope on expected utility of the uninformed adviser is more negative than the slope on expected utility of the informed adviser at every $\omega > \hat{\omega}$. Thus, the first-period costs of using information too aggressively are higher for the uninformed adviser than for the informed adviser. This allows the informed adviser to use his first-period portfolio choice to signal his superior information to investors.

The interactions among the precision of the informed adviser's information, the size of the asset fee, and risk tolerance are complex for $\alpha \in (-1, \frac{1}{2}]$. The case of logarithmic utility, corresponding to $\alpha = 0$, is illustrative. It can be shown that when $f > 1 - (1/\sqrt{2})$, the only equilibrium is the separating equilibrium described in Proposition 2. More generally, increases in risk aversion or the asset fee make separation easier to support. Increases in the precision of the informed adviser's information (i) reduce the set of parameter values α and f for which the informed adviser must distort his portfolio choice to achieve separation and (ii) reduce the magnitude of the distortion when a distortion is necessary.¹⁶

¹⁶This follows because the right-hand side of Eq. (5) decreases in ω and increases p .

2.4. *Effort aversion in information acquisition*

As noted in the introduction, the model presented above suppresses agency costs associated with motivating advisers to seek out costly information to improve portfolio choice. This section explores the implications of incorporating effort aversion into the analysis. Perhaps the simplest generalization of the model to account for effort aversion is to assume the uninformed adviser cannot receive a signal regardless of his effort choice, while the probability an informed adviser's information is correct increases in effort e , so that $p = \frac{1}{2} + e$, and effort reduces adviser utility by an additively separable quantity $c(e)$ where $c(0) = 0$, $c(\frac{1}{2}) = \infty$, and c is monotone. The situation studied in this paper corresponds to a limiting specification of c : $c(e) = 0$ for $e \leq \bar{p} - \frac{1}{2}$ and $c(e) = \infty$ otherwise. For $c(e)$ sufficiently close to the limiting specification, the results reported above describing conditions under which pooling and separating equilibria obtain would be qualitatively similar, since the identifying inequalities in the proofs are strict. Comparative statics for continuous, differentiable $c(e)$ do not appear tractable, though one can speculate that exerting more effort to acquire a more precise signal and choosing a more extreme portfolio weight are substitutes in achieving separation. For this reason, pooling should obtain for a larger set of exogenous parameters as the cost of effort for the informed adviser increases.

2.5. *Endogenizing fees*

To this point in the paper, fees have been exogenous and the number of advisers has been fixed at two. This section explores how equilibrium fees are set assuming free entry and a fixed cost of operating a mutual fund. To keep the analysis tractable, suppose there is a single informed adviser and a number of uninformed agents who may choose to open funds and become advisers by paying a cost c . Each agent who pays c makes a take-it-or-leave-it offer to manage investors' funds for an asset fee f levied after returns are realized each period. Agents who do not pay c earn their reservation utility, $\bar{U}$. If an investor accepts the offer, she places all her wealth with a single adviser. An investor who rejects all offers invests in the market portfolio, M , which has a return normalized to zero. From the investors' perspective, in the first-best outcome, no uninformed agents become advisers.

The informed adviser cannot use his choice of fee f as a signaling device because, conditional on having paid c , it is costless for the uninformed advisers to make the same fee choice as the informed adviser. Any uninformed adviser who offers a fee different from the equilibrium fee charged by the informed adviser will thereby reveal himself to be uninformed. The payoff to investors from investing with an uninformed adviser who charges an asset fee $f > 0$ is less than the return on investing directly in the market portfolio. Hence, an adviser

who reveals himself to be uninformed by his choice of asset fee receives no funds to manage in either trading round and is out of pocket c . Thus, every agent who becomes an adviser charges the same fee.

Since advisers make take-it-or-leave-it offers, the fee they charge leaves investors just indifferent between placing their wealth with an adviser to invest or investing in the next best alternative, namely the market portfolio. Call this the ‘break-even fee’. Further assume that investors believe any adviser who offers a fee different from the break-even fee is surely uninformed.¹⁷ The game then proceeds as outlined in Table 1.

2.5.1. Separating equilibrium

Let n be the number of uninformed agents who become advisers. Three conditions must hold in a separating equilibrium: First,

$$U(1) = \frac{1}{n+1} [pU((1-f)(1+\hat{\omega})) + (1-p)U((1-f)(1-\hat{\omega})) + nU(1-f)]Q, \quad (6)$$

where $Q = p((1-f)(1+\hat{\omega}))^2 + (1-p)((1-f)(1-\hat{\omega}))^2$, the contribution to investors’ expected utility from investing with the informed adviser in period 2 and $\hat{\omega} = \max(\hat{\omega}, \bar{\omega})$, as in Proposition 2; second,

$$\begin{aligned} & \frac{p}{2} U \left(f \left[\frac{1}{n+1} (1 + \bar{\omega}) + \frac{(1-f)}{2} \left(\frac{2}{n+1} (1 + \bar{\omega}) + \frac{n-1}{n+1} \right) \right] \right) \\ & + \frac{p}{2} U \left(f \left[\frac{1}{n+1} (1 - \bar{\omega}) + 0 \right] \right) + \frac{1-p}{2} U \left(f \left[\frac{1}{n+1} (1 - \bar{\omega}) \right. \right. \\ & \left. \left. + \frac{(1-f)}{2} \left(\frac{2}{n+1} (1 - \bar{\omega}) + \frac{n-1}{n+1} \right) \right] \right) + \frac{1-p}{2} U \left(f \left[\frac{1}{n+1} (1 + \bar{\omega}) \right. \right. \\ & \left. \left. + (1-f) \left(\frac{1}{n+1} (1 + \bar{\omega}) + \frac{1}{n+1} (1 - \bar{\omega}) + \frac{n-1}{n+1} \right) \right] \right) = U \left(\frac{f}{n+1} \right); \quad (7) \end{aligned}$$

and, third,

$$U \left(\frac{f_{n+1}}{n+2} \right) - c < \bar{U} \leq U \left(\frac{f_n}{n+1} \right) - c$$

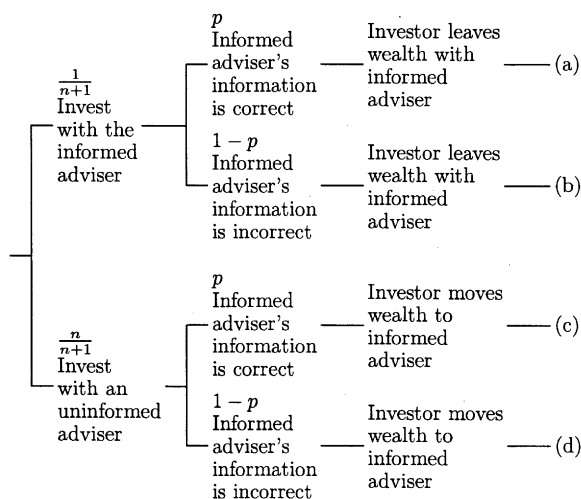
or

$$U \left(\frac{f_{n+1}}{n+2} \right) < \bar{U} + c \leq U \left(\frac{f_n}{n+1} \right). \quad (8)$$

¹⁷ Investor beliefs must be specified to establish an equilibrium fee. Holding fixed the number of agents who become advisers, these beliefs provide the highest fee to advisers.

In Eq. (8), the subscript on f makes explicit the dependence of the fee on the number of uninformed traders.

Condition (6) says investors are just indifferent to investing directly in the market portfolio, M , and investing in a mutual fund given a separating equilibrium where there are n uninformed advisers and a single informed adviser. The right-hand side of this expression follows from the inference tree in Fig. 1. Condition (7), a generalization of the expression $M(\omega)$ used in the proof of Proposition 1, guarantees that no uninformed adviser will deviate from the separating equilibrium by mimicking the informed adviser since the payoff from doing so does not exceed the payoff from revealing himself to be uninformed



Event	Probability	Payoff
(a)	$\frac{p}{n+1}$	$U((1-f)(1+\hat{\omega}))Q$
(b)	$\frac{1-p}{n+1}$	$U((1-f)(1-\hat{\omega}))Q$
(c)	$\frac{p}{n+1}$	$U((1-f))Q$
(d)	$\frac{1-p}{n+1}$	$U((1-f))Q$

Fig. 1. This figure depicts the possible outcomes for an investor who places her wealth with a randomly chosen adviser in period 1 and who may shift her wealth to another adviser at the end of that period given (i) there are n uninformed advisers and one informed adviser, and (ii) the equilibrium is separating.

after the first round by choosing $\omega_0 = 0$. Finally, condition (8) links the cost of entry c by uninformed advisers to the utility each of those advisers receives in a separating equilibrium.

While these relationships are complex, several observations can be made about the structure of the problem for general c . Provided $f_n/(n+1)$ is decreasing in n , it is plain from Eq. (8) that decreasing the entry cost, c , will increase the equilibrium number of uninformed advisers. Panel A of Table 3 presents numerically-determined values of the asset fee, f , the first period portfolio weight, $\bar{\omega}$, chosen by the informed adviser, and the payoff to an uninformed adviser (excluding the payment of c) that are implied by Eqs. (6) and (7) for $\alpha = -1$ and $p = 0.9$ and various values of n . Since the uninformed advisers' payoffs are declining in n , more agents become uninformed advisers as c falls. For instance, for $15.7 \leq \bar{U} + c < 16.9$, Eq. (8) implies one agent will become an adviser. Thus, the two-adviser case considered earlier fits within this framework. To achieve separation, the informed adviser must make more extreme portfolio choices $\bar{\omega}$ as more agents enter. Finally, while the asset fee f decreases in n in this case, the property is not general. In panel B of Table 3, the asset fee increases as the number of uninformed advisers increases. To see why this is so, observe that if $\bar{\omega}$ is extreme given the informed adviser's information so that mimicry by the uninformed advisers is discouraged, then investors may prefer entry by more uninformed advisers. This reduces the prior probability an investor will invest with the informed adviser in period 1 when that adviser makes an extreme, hence risky and costly, portfolio choice. Since the equilibrium is separating, increasing the number of uninformed advisers does not prevent the investor from investing with the informed adviser for sure in the second period. Only a finite number of advisers enter since their utility decreases even as the asset fee increases due to the smaller amount of assets each manages in the first period.

2.5.2. Pooling equilibrium

Assuming the same structure of take-it-or-leave-it offers that precede the game laid out in Table 1 as for the separating case, there are a set of conditions that must be satisfied in a pooling equilibrium defined by strategies $s_I((\hat{\omega}, \check{\omega}); (1, 0), (0, 1))$ for the informed adviser and $s_U((\check{\omega}, \hat{\omega}); (\frac{1}{2}, \frac{1}{2}))$ for each of n uninformed advisers. Relative to the case of one informed and one uninformed adviser, more structure must be imposed to specify advisers' payoffs in the pooling equilibrium when there are more advisers. This structure specifies how investors who invested with advisers that underperformed in the first period reallocate their wealth among the advisers who outperformed. Again, to keep the analysis simple, suppose that each investor reallocates all her wealth to an adviser who outperformed; and, in aggregate, investors reallocate their wealth equally across advisers who outperform. Advisers who underperform manage none of investors wealth in the second period unless all advisers underperform,

Table 3

Equilibrium asset fee, informed adviser's portfolio weight in the separating equilibrium

This table shows the equilibrium fee, $\bar{\omega}$, informed adviser's first period portfolio weight, $\bar{\omega}$, and the payoff to an uninformed adviser (excluding the payment of c), for various numbers of uninformed advisers, n in a separating equilibrium. The other parameter is the coefficient of risk aversion, $\alpha = -1$.

	n	f	$\bar{\omega}$	Uninformed adviser's payoff
Panel A: $p = 0.90$	1	0.127	0.741	– 15.7
	2	0.118	0.751	– 16.9
	3	0.114	0.756	– 17.5
	5	0.111	0.760	– 18.0
	10	0.108	0.766	– 18.5
	50	0.106	0.772	– 481.0
	100	0.106	0.774	– 954.6
	1000	0.105	0.775	– 9,479.7
Panel B: $p = 0.85$	1	0.055	0.749	– 36.3
	2	0.057	0.762	– 35.1
	3	0.060	0.768	– 33.5
	5	0.063	0.760	– 31.5
	10	0.068	0.778	– 29.5
	50	0.073	0.785	– 702.1
	100	0.073	0.786	– 1,375.9
	1000	0.074	0.787	– 13,502.4

in which case each adviser retains the funds he managed at the end of the first period.

Analogous to Eq. (6) is the condition that investors be just indifferent to investing directly in the market portfolio, M , and investing in a mutual fund given a pooling equilibrium where there are n uninformed advisers and a single informed adviser. The right-hand side of this expression follows from the inference tree in Fig. 2.

$$\begin{aligned}
 U(1) = \frac{1}{n+1} & \left[pU((1-f)(1+\hat{\omega}))Q + \left(\frac{p}{2} + \frac{1-p}{2^n} \right) U((1-f)(1-\hat{\omega}))Q \right. \\
 & + \frac{n+2-3p-2^{1-n}(1-p)}{2} U((1-f)((1-\hat{\omega}))(1-f)^z) \\
 & \left. + \frac{n}{2} U((1-f)(1+\hat{\omega}))(1-f)^z \right]. \quad (9)
 \end{aligned}$$

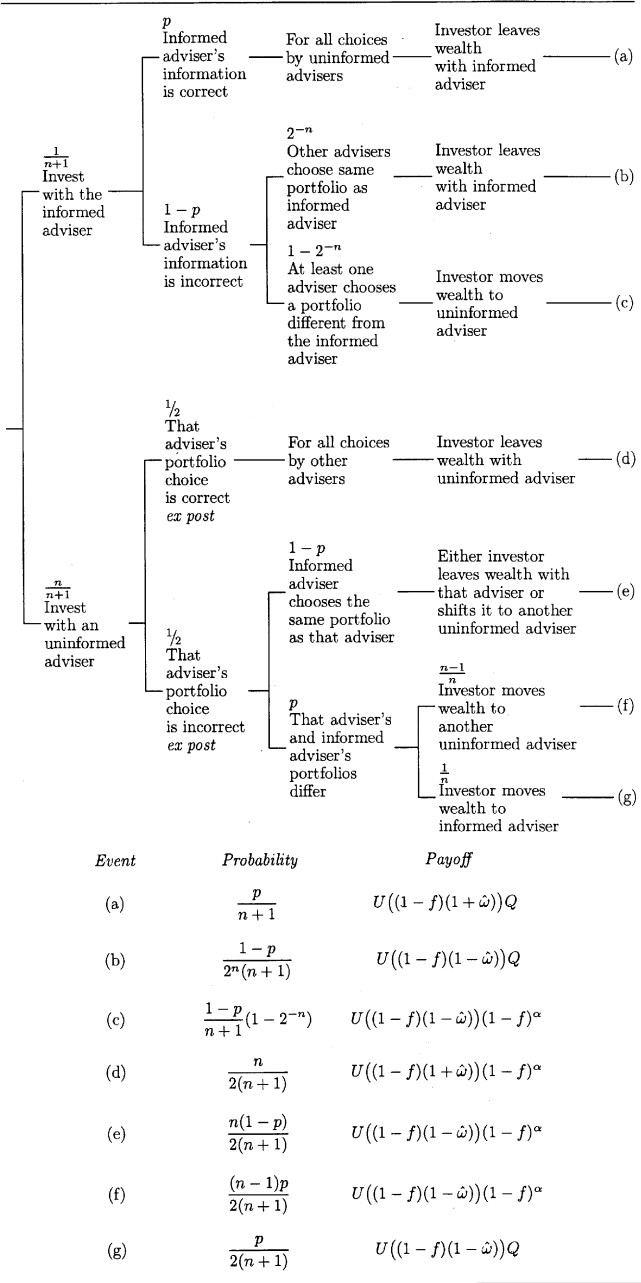


Fig. 2. This figure depicts the possible outcomes for an investor who places her wealth with a randomly chosen adviser in period 1 and who may shift her wealth to another adviser at the end of that period given (i) there are n uninformed advisers and one informed adviser, and (ii) the equilibrium is pooling so that $s_I((\hat{\omega}, \hat{\omega}); (1, 0), (0, 1))$ for the informed adviser and $s_U((\hat{\omega}, \hat{\omega}); (\frac{1}{2}, \frac{1}{2}))$ for every uninformed adviser.

Given $n + 1$ advisers $i \leq n$ of which underperform, each of the $n - i + 1$ advisers that outperform receive incremental funds to manage of

$$\frac{i}{n - i + 1}(1 - f)(1 - \hat{\omega}).$$

Conditional on a given uninformed adviser outperforming, the probability of exactly i other investors underperforming is

$$2^{1-n} \left[p \binom{n-1}{i} + (1-p) \binom{n-1}{i-1} \right].$$

When an adviser chooses a portfolio consistent with being informed and is revealed *ex post* to have outperformed, then that adviser retains in the second period all the funds he managed in the first period. The payoff to an uninformed adviser in the pooling equilibrium (excluding the cost c) is

$$\begin{aligned} EU_U(f, p, \alpha; n) = & \frac{1}{2} \sum_{i=0}^n 2^{1-n} \left[p \binom{n-1}{i} + (1-p) \binom{n-1}{i-1} \right] U \left(f \left(\frac{(1+\hat{\omega})(2-f)}{n+1} \right. \right. \\ & \left. \left. + \frac{i(1-f)(1-\hat{\omega})}{(n+1)(n-i+1)} \right) \right) + \frac{1}{2} \left[\left(1 - \frac{1-p}{2^{n-1}} \right) U \left(f \frac{1-\hat{\omega}}{n+1} \right) \right. \\ & \left. + \frac{1-p}{2^{n-1}} U \left(f \left(\frac{1-\hat{\omega}}{n+1} + (1-f) \frac{1-\hat{\omega}}{n+1} \right) \right) \right]. \end{aligned}$$

To sustain the equilibrium, each uninformed adviser must prefer this payoff to the payoff he would receive from revealing himself to be uninformed in the first round, i.e., $U(f/(n+1))$. It is straightforward to verify that this condition holds strictly at $\alpha = 1$ (i.e., risk neutrality) for any $n \in \mathbb{N}$ and $f \in (0, 1)$, hence it also holds in a neighborhood of $\alpha = 1$. Analogous to Eq. (8), the number of uninformed advisers, n follows from the condition that

$$EU_U(f_{n+1}, p, \alpha; n+1) < \bar{U} + c \leq EU_U(f_n, p, \alpha; n). \quad (10)$$

Table 4 presents numerically-determined values of the asset fee, f , implied by Eq. (9) and the payoff to the uninformed adviser (excluding the payment of c), $EU_U(f_n, p, \alpha; n)$, for $\alpha = 0.95$ and $p = 0.95$. These payoffs exceed $U(f/(n+1))$. Since the payoff decreases in n , it follows from (10) that the number of advisers increases as entry costs fall in this case. As n grows large, (i) investors are more likely to be invested with an uninformed adviser, who exposes them to risky portfolio choices; and, (ii) as it is more likely that several uninformed advisers will outperform the market in the first period, investors are less likely to reallocate their wealth to the informed adviser. Both effects diminish the attractiveness of investing in mutual funds. Consequently, the equilibrium asset fee, f , falls sharply in the number of uninformed advisers – from 29.2% for $n = 1$ to

Table 4

Equilibrium asset fee in the pooling equilibrium

This table shows the equilibrium fee, f , and the payoff to an uninformed adviser (excluding the payment of c), for various numbers of uninformed advisers, n in a pooling equilibrium where the informed adviser's strategy is $s_I((\hat{\omega}, \hat{\omega}); (1, 0), (0, 1))$. The other parameters are $\alpha = 0.95$ and $p = 0.95$.

n	f	Uninformed adviser's payoff
1	0.292	0.2716
2	0.142	0.1009
3	0.012	0.0078

1.2% for $n = 3$. For the example given, investors prefer to invest passively rather than through mutual funds for $n > 3$. This is an additional constraint, besides the cost of entry, on the number of advisers.

2.5.3. Calibration of the model

It is interesting to ask how the model behaves for parameters that approximate actual market contracts and outcomes. The probability that an informed adviser outperforms the market, p , is unlikely to be more than 65%. Assuming a separating equilibrium, $p = 65\%$, and a risk aversion parameter of $\alpha = -1$, numerical investigation shows that the asset fee increases as the number of uninformed advisers increases. The number of uninformed advisers in a separating equilibrium must be at least 469 for the equilibrium asset fee to exceed 1% per period. For values of $\alpha < -1$ even larger numbers of uninformed advisers are required before the asset fee exceeds 1%. This suggests that the ratio of informed to uninformed advisers must be quite small for the separating equilibrium to obtain for plausible values of p . In this equilibrium, most investors would invest with an adviser who claims to be informed, but reveals himself after one round of trading to have held the market portfolio and hence to be uninformed. In the second period, all investors would flock to the adviser who chose a portfolio different from the market portfolio and who thereby reveals himself to be informed.

Assuming a pooling equilibrium, $p = 65\%$, and a risk-aversion parameter of $\alpha = 0.95$, numerical investigation shows that the asset fee decreases as the number of uninformed advisers increases. When there is one informed and one uninformed adviser, the equilibrium asset fee is 1.5%. There is no positive asset fee that would induce investors to invest with an adviser chosen at random from a pool containing two or more uninformed advisers but only one informed adviser. This suggests the number of informed advisers in a pooling equilibrium approximates or exceeds the number of uninformed advisers. In this equilibrium, investors would invest with roughly equal probability with an informed or uninformed adviser. Since all advisers choose portfolios that differ from the

market portfolio, none are revealed to be uninformed with certainty, but it is more likely that the informed advisers outperformed, and hence investors would reallocate their wealth to those advisers.

2.6. *Effect on Portfolio Risk*

In either the pooling or separating equilibrium, one adviser is using his information too aggressively. This increases the variance of mutual fund returns.

Proposition 3. The variance, whether or not conditioned on the signal, of fund returns is higher in the presence of a reputation effect than in a single-period setting where there is no reputation effect.

An investor cannot hedge this risk by holding some quantity of the risky asset directly because she does not know whether she is invested with the informed or uninformed adviser. Moreover, the investor does not have the informed adviser's information and the uninformed adviser is playing a mixed strategy, so the investor does not know whether to go long or short in the risky asset to hedge the exposure due to the reputation effect. Holding some of the market portfolio directly can serve to hedge a part of this risk, but transactions cost may be very high. Section 3 shows how performance fees can reduce the risk induced by reputation concerns and enhance welfare.

2.7. *New money*

The basic tension in this paper arises because a prize available tomorrow (i.e., additional funds to manage) depends on whether a risky bet (i.e., a portfolio choice that signals ability) is taken today. Making no bet means no probability of winning the prize. Making a bet imposes risk that reduces utility in the first period but raises expected payout in the second period. If the prize is big in relation to the risk, then the uninformed adviser will take the bet. In the model, the prize comes from money leaving one fund to enter another. This structure is appealing because it creates a closed system, though the same intuition applies to 'new money' as well. Suppose investors leave their period one investments with the same adviser in period two regardless of performance.¹⁸ Also suppose some new money will be invested with the advisers at the start of period two. If the allocation of new money between the advisers is sensitive to first period performance, then the uninformed adviser will wish to mimic the informed adviser while the informed adviser will wish to separate himself from the

¹⁸ Patel et al. (1992) document that old money is slow to leave established funds with poor track records but new money flows to funds with good track records.

uninformed adviser. Whether pooling or separation obtains depends on the amount of new money invested in period two relative to the amount of money invested in period one as well as the precision of the informed adviser's information and the risk tolerance of the advisers.

Both informed and uninformed investment advisers are rewarded for taking on risk, but for different reasons. It is rational for an uninformed adviser to overinvest in risky assets in order to appear informed; this gives rise to a pooling equilibrium. On the other hand, an informed adviser may benefit from an even more aggressive asset positions in risky assets in order to separate by increasing the cost of mimicry for the uninformed adviser.

When advisers are nearly risk neutral and asset fees are small, only pooling equilibria are possible since the cost to an uninformed adviser of choosing a risky portfolio is reduced. Also, separation becomes cheaper for the informed adviser and mimicry becomes more expensive for the uninformed adviser as the precision of the informed adviser's information increases.

3. Performance fees

Performance fees reward (punish) advisers when the return on the portfolio they manage is greater (less) than the return on a benchmark portfolio. This section examines the impact of linear performance fees on the portfolio choices made by advisers. It also considers the consequent effects on investors. A linear performance fee is a constant g times the difference between the return on the adviser's portfolio and the return on the benchmark portfolio.¹⁹ Such a fee induces a Pareto improvement when advisers make portfolio choices consistent with the separating equilibrium described in the previous section. It also increases ex ante investor wealth under the pooling equilibrium.

Contracts for investment advisory services are regulated in the United States under the Investment Advisers Act of 1940. Since 1970, Section 205(a)1 of the Act prohibits a registered investment adviser from receiving compensation 'on the basis of a share of capital gains upon or capital appreciation of the funds or any portion of the funds of the client', including clients who are registered investment companies (i.e., mutual funds). The reason for this prohibition is Congress' concern that such fees would induce advisers to take inappropriate risks that would harm investors [Division of Investment Management (1992),

¹⁹ Some performance contracts stipulate a minimum fee that will be paid to the adviser regardless of performance. This fee floor would cause the adviser's compensation to resemble a call option on a basket of risky assets if it were likely the portfolio return fall below the threshold. In practice, the fee floors in many contracts are unlikely to be reached even with mediocre portfolio performance (Davanzo and Nesbitt, 1987; Grinold and Rudd, 1987; Record and Tynan, 1987). In such cases, the linear approximation is reasonable.

pp. 237–238]. One exception to this rule is a ‘fulcrum fee’. With a fulcrum fee, increases in an adviser’s compensation for performance in excess of a benchmark equal decreases in compensation for performance that falls short of the benchmark by a like amount.

One reason the SEC insists on fulcrum fees is the fear that other types of fees cause advisers to make speculative investments. In such cases, the adviser wins a great deal if the investment turns out well because he participates in the upside of the transaction. If the investment turns out badly, the adviser is not greatly harmed because his downside risk is limited. Also, if fees were not symmetric then an adviser would have an incentive to set up multiple funds (Kritzman, 1987). In one fund he could buy, and in the other sell, the same asset. One of these strategies is bound to be profitable while the other will not be. If fees are not symmetric, then the adviser is rewarded with a performance bonus in the fund that performed well, but there is no offsetting penalty in the fund with the opposite position. The linear fees considered below do not suffer from this criticism.

Remark 2. Relative to the case of no performance fees, linear performance fees induce advisers to use their information less aggressively.

Suppose that a performance fee of g is imposed and the benchmark portfolio is M . The payoff to the adviser who assesses a probability q that the risky asset $\tilde{A}$ will be worth 2 next period and chooses portfolio ω is

$$\begin{aligned} & EU(f(\omega\tilde{A} + (1 - \omega)M) + g(\omega\tilde{A} + (1 - \omega)M - M)) \\ &= qU(f(1 + \omega) + g\omega) + (1 - q)U(f(1 - \omega) - g\omega). \end{aligned}$$

The effect of imposing a performance fee in a one-period setting is shown in Fig. 3, which presents four plots an adviser’s expected utility in a one-period setting as a function of the portfolio weight on asset $\tilde{A}$. The four curves correspond to (i) uninformed adviser, no performance fee, (ii) uninformed adviser with performance fee, (iii) informed adviser, no performance fee, and (iv) informed adviser with performance fee. The informed adviser is assumed to receive the signal G . The cases corresponding to signal B are symmetric. For the uninformed adviser, all portfolios offer the same expected return because the uninformed adviser believes asset $\tilde{A}$ pays either 0 or 2 with equal probability. Since the portfolio $\omega_0 = 0$ is riskless, it maximizes the uninformed adviser’s utility. The expected utility of the uninformed adviser is lower the more extreme the position in asset $\tilde{A}$. The maximum utility and optimal portfolio choice of the uninformed adviser do not change when a performance fee applies. The difference between the expected one-period payoff from holding the optimal portfolio, $\omega_0 = 0$, and the payoff from holding any other portfolio, ω , increases with the imposition of a performance fee. The larger the performance fee g , the greater the

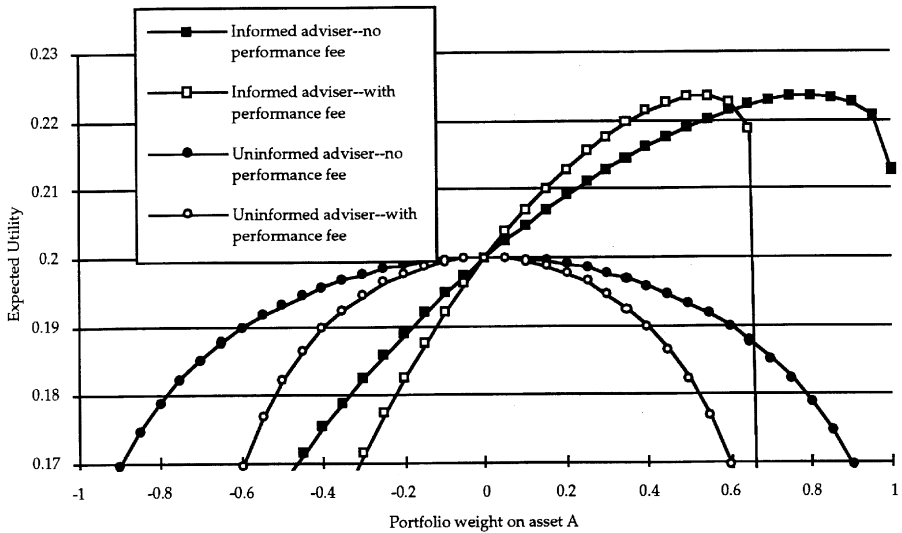


Fig. 3. One-period payoff to informed and uninformed advisers as a function of portfolio choice. This figure plots the one-period payoffs to the uninformed and informed advisers under two different scenarios. In the first scenario, an asset fee is paid to each adviser. No performance fee is paid. In the second scenario, an asset fee and a symmetric performance fee are paid to each adviser. Performance is measured relative to the market portfolio. Parameter values are: coefficient of risk aversion, $\alpha = \frac{1}{2}$; probability asset $\tilde{A}$ will be worth 2 at the end of next period, $p = \frac{3}{4}$; asset fee, $f = 1\%$; performance fee, $g = 0.5\%$. The figure assumes the informed adviser receives the signal G . If the informed adviser receives the signal B , the curves are reflected in the vertical axis through zero.

difference. The performance fee penalizes an adviser for choosing a portfolio other than the benchmark portfolio by increasing risk as the chosen portfolio departs from the benchmark portfolio.

For the informed adviser, both returns and risk differ across portfolios. For the parameters underlying Fig. 3, the portfolio chosen by the informed adviser is about 80% risky asset $\tilde{A}$ and 20% benchmark portfolio M when there is no performance fee (i.e., $g = 0$). When a performance fee of 0.5% is imposed, the informed adviser's optimal portfolio choice shifts to about 50% risky asset and 50% benchmark portfolio. He receives exactly the same expected utility from this portfolio choice when the performance fee applies as he would with 80% of his funds invested in the risky asset when no performance fee applies. As the performance fee increases, the manager is willing to bet less, because the risk imposed by the performance fee increases. The optimal one-period portfolio when performance fees are imposed lies closer to the benchmark portfolio than when no performance fee is imposed. So the performance fee causes the informed adviser to hold a portfolio that is a linear combination of the benchmark portfolio and the portfolio he would choose in the absence of a performance fee.

It is easy to envision the effect of varying the composition of the benchmark and the sign and magnitude of the performance fee, g , on the portfolio chosen by the informed adviser. Recall there are two assets in the economy, $\tilde{A}$ and M . Feasible portfolio weights are ordered pairs $(\omega, 1 - \omega)$. Call $\omega^*(0)$ the portfolio weight on asset $\tilde{A}$ chosen by the adviser in the absence of a performance fee. Call λ the weight on asset $\tilde{A}$ in the benchmark portfolio whose return is subtracted from the return on the adviser's portfolio in computing the performance fee. In this paper, $\lambda = 0$. Each point on the line through $(\omega^*(0), 1 - \omega^*(0))$ and $(\lambda, 1 - \lambda)$ in $\mathbb{R}^2$ corresponds to the optimal portfolio choice for a particular performance fee, g . As g increases from zero, the portfolio chosen by the adviser moves along the line from $(\omega^*(0), 1 - \omega^*(0))$ toward the benchmark portfolio, $(\lambda, 1 - \lambda)$.²⁰

Fig. 3 also suggests that the maximum expected utility derived by the adviser when no performance fee is imposed is the same as the maximum expected utility, derived from a *different* portfolio, when a performance fee is imposed. Proposition 4 confirms this.

Proposition 4. Define

$$\omega(g) = \frac{f\omega(0) + g\lambda}{f + g}. \quad (11)$$

*The one-period payoff to an adviser in the absence of performance fees when portfolio $(\omega(0), 1 - \omega(0))$ is chosen is identical to the payoff when performance fee g is imposed and portfolio $(\omega(g), 1 - \omega(g))$ is chosen.*²¹

The proof follows immediately from replacing $\omega(g)$ with the right-hand side of Eq. (11) in

$$\begin{aligned} & f(\omega(g)\tilde{A} + (1 - \omega(g))M) + g(\omega(g)\tilde{A} + (1 - \omega(g))M - \lambda\tilde{A} - (1 - \lambda)M) \\ & = f(\omega(0)\tilde{A} + (1 - \omega(0))M). \end{aligned} \quad (12)$$

An immediate and surprising implication of Proposition 4 is that linear performance fees provide no incentives to acquire costly information about asset

²⁰ As g decreases in the range $-f < g < 0$, the portfolio chosen by the manager moves along the line away from $(\lambda, 1 - \lambda)$ beyond $(\omega^*(0), 1 - \omega^*(0))$. As g decreases in the range $g < -f$, the portfolio chosen by the manager approaches $(\lambda, 1 - \lambda)$ from the side opposite $(\omega^*(0), 1 - \omega^*(0))$.

²¹ Proposition 4 generalizes naturally to an economy where there are many assets. If there are n assets, the result holds when ω and λ are interpreted as n -dimensional vectors. The i th component of each vector is the weight on asset i . Of course, the elements in the vectors are restricted to sum to one.

returns.²² Performance fee g only causes the adviser to alter her portfolio choice. Where effort to acquire information is costly, as outlined earlier, the asset fee and the reputation effect motivate information acquisition, but a performance fee in no way augments these. A further implication is that if (i) the adviser has no wealth outside the fund he manages, and (ii) infinite marginal utility for wealth at zero, then a performance fee can never make it worthwhile for an adviser to offer a negative asset fee, even if a negative asset fee would allow an adviser to signal his type. Lemma 5 in the appendix proves this.

If advisers do have wealth that is not invested in the fund they manage, then an informed adviser might accept a zero or negative asset fee, provided a performance fee is also offered. The adviser can use his private information to choose a portfolio for the fund such that the performance fee increases the adviser's expected utility. Because the performance fee payment cannot increase the expected utility of an uninformed adviser, an uninformed adviser would not accept a zero or negative asset fee. Provided the asset fee is non-positive in each period, there is no benefit to the uninformed adviser from appearing informed in the first period. Hence, non-positive asset fees may be used to screen for informed advisers in this case.

Proposition 4 also shows that the compensation received by an adviser is independent of the choice of benchmark or the magnitude of the performance fee. For any portfolio chosen by the adviser in a regime where there is no performance fee, there corresponds a related portfolio defined by Eq. (11) that provides the same compensation *state-by-state* when performance fee g is imposed. Returning to the case where all of an adviser's wealth invested in the fund he manages and asset fees are positive, the utility-maximizing compensation is the same regardless of the benchmark and performance fee imposed. Furthermore, if each investment adviser faces the same asset and performance fees then the difference in the expected utilities of the advisers when both choose $\omega(g)$ is exactly the same as when no performance fee is imposed and both choose $\omega(0)$ for any ω and g . Thus, an equilibrium at $\omega^*(0)$ when $g = 0$ is also an equilibrium at $\omega^*(g)$ for any given g . This observation lies at the heart of the next two propositions.

The analysis in the paper is a special case in which there is no interaction between the portfolio choices of any adviser and the returns on either the benchmark, the market portfolio, or the risky asset about which the informed adviser has private information. This is admittedly an extreme situation, but Proposition 4 is neat in this context. Proposition 4 continues to hold if the actions of advisers affect stock returns provided the portfolio weights in the benchmark portfolio are known and fixed over the period during which the performance fee is calculated. Alternatively, the benchmark portfolio weights

²² Stoughton (1993) makes a similar observation.

may not be known to the adviser, because, for instance, the benchmark is the sum of the portfolios simultaneously chosen by other advisers. In this case, the Proposition 4 does not hold since the adviser must replace λ , now a random variable, in (11) with some estimate such as $E[\hat{\lambda}]$. Then Eq. (12) holds only in expectation and not state-by-state. Since the portfolio weights are risky, the adviser's expected utility from choosing $\omega(g)$ (using estimates of the weights) when performance fee g is imposed is less than her utility from choosing $\omega(0)$ when no performance fee is imposed. But if the benchmark is a peer group of advisers who in aggregate hold a portfolio with composition and returns that are very similar to an unmanaged, predictable index, then Proposition 4 will hold approximately.²³ When the exact composition of the benchmark portfolio is unknown to the adviser, it is still the case that if the adviser holds a portfolio that more closely resembles what she expects the benchmark to be, the risk she bears from the imposition of the performance fee is reduced, and so the manager will tilt her portfolio choices toward the (expected) benchmark.

Why are performance fees imposed at all? One answer to this question is that the portfolio selected by the adviser changes as a function of the benchmark and performance fee. A benchmark and performance fee can serve as tools to modify the portfolio choices of advisers. In particular, the performance fee and benchmark can be chosen to counteract the undesirable distortions in portfolio choice induced by reputation concerns.

Suppose the adviser's choices are given by the separating equilibrium identified in Proposition 2. Also, suppose that $\hat{\omega} < \hat{\omega}$. If performance fees are adopted, the informed adviser chooses a portfolio that is closer to $\hat{\omega}$ than $\hat{\omega}$. Separation is maintained because it is as costly for the uninformed adviser to mimic $\hat{\omega}$ in the absence of a performance fee as it is to mimic $\hat{\omega}(g)$ in the presence of performance fee g . This means the uninformed adviser will choose $\omega_0 = 0$ for any performance fee g . Happily, this is the portfolio choice the investors prefer him to make. From Proposition 4, the expected utility to each adviser is the same regardless of the asset fee, g , but the utility of the investors can be improved by judicious choice of g , so a Pareto improvement is possible. Holding f fixed, the best choice of g affords investors the utility they would obtain in the absence of reputation concerns.

Proposition 5. To every separating equilibrium identified by Proposition 2, there corresponds a set of separating equilibria indexed by the performance fee, g . The expected utilities of advisers are constant over this set. There is a unique performance fee imposed in the first period that provides investors with the expected utilities they would obtain in the absence of reputation concerns.

²³ It seems plausible that the composition of the S&P 500 index is so well publicized that one would expect the undoing result to apply, although other indices may be harder to track.

Imposing the optimal performance fee g does not require investors to know which adviser is informed or the information of the informed adviser. A performance fee alters the equilibrium portfolio choices made by the informed adviser without altering the separating nature of the equilibrium. Increasing g shifts investor wealth, unit for unit, from the state where the adviser's private information turns out to be correct to the state where the adviser's private information turns out to be incorrect. This lowers the expected value of the portfolio – since $p > 1/2$, the adviser is more likely to be right than wrong – but makes the payoff less risky. Investors are made better off by reducing both risk and return.

Both types of advisers find it optimal to choose a portfolio closer to the market portfolio when a performance fee applies. The total risk assumed by the investors and adviser together is less when a performance fee is imposed. If advisers choose portfolios consistent with the pooling equilibria identified in Proposition 1, then performance fees increase the welfare of investors who invested with the uninformed adviser, but may decrease the welfare of investors who invested with the informed adviser. Imposing a performance fee moves the uninformed adviser's portfolio choice closer to his one-period optimum, but moves the informed adviser's portfolio choice, ω_i , further from the one-period optimum whenever $|\omega_i| \leq \hat{\omega}$. The effect of a judiciously chosen performance fee on the expected utility of investors under the pooling equilibrium is positive, but the effects of reputation considerations cannot be eliminated in this case.

*Proposition 6. To every pooling equilibrium identified by Proposition 1, there corresponds a set of pooling equilibria indexed by the performance fee, g . The expected utilities of advisers are constant over this set. There is a performance fee imposed in the first period that improves the expected utilities of investors relative to the case of no performance fees.*²⁴

Linear performance fees induce advisers to alter their portfolio choices in a way that changes investors' payoffs but does not alter the nature of the equilibrium. Moreover, the payments to advisers *state-by-state* are unchanged. Thus, Propositions 4–6 apply whatever the information structure. If effort to acquire information were costly, these results would still apply whatever the equilibrium effort choice.

Imposing a performance fee causes an adviser to choose a portfolio that more closely resembles the benchmark portfolio. This is welfare improving when it offsets reputation considerations that cause an adviser to choose a portfolio that

²⁴ Earlier in the paper it was assumed that each investor is as likely to invest with the informed adviser as with the uninformed adviser in period one. That assumption simplifies the analysis in Section 1 but is not required in the proofs of Propositions 5 and 6, which only require that investors (i) are uncertain whether they are invested with informed or uninformed adviser and (ii) share the same beliefs about this uncertainty.

is too extreme. There are alternative means for investors to achieve the same result. If investors can buy directly the assets underlying the fund, then by dividing their wealth between the assets underlying the benchmark and a fund without a performance fee, they can achieve a combined portfolio with the same composition as a fund with a performance fee. Once transaction costs are introduced however, it becomes important to consider whether a performance fee or direct investment in the underlying assets is the cheaper way to mitigate the reputational distortions in portfolio choice. Imposition of a performance fee requires the rewriting of a single contract between the adviser and the fund. The alternative involves obliging each investor to take a position in the benchmark portfolio. Even if a traded index that represents the benchmark portfolio exists, every investor will need to buy this index. Particularly in the cases of (i) many investors with small amounts of wealth to invest, (ii) assets that are difficult to buy directly (e.g., foreign stocks), or (iii) benchmark portfolios that are not tradable (e.g., real estate), performance fees may be the most effective mechanism to adjust investors' portfolios.

4. Conclusion

The interactions among reputation effects, management fees, risk aversion, precision of private information, investor welfare, and entry in the investment advisory industry are complex. This paper analyzes how different types of fees affect distortions due to reputational considerations under risk aversion.

In large part, the form of investment advisers' compensation contracts is exogenously imposed by the Securities and Exchange Commission. This paper asks, given the contractual forms that closely resemble those imposed by the SEC, what behavior emerges when advisers rationally cultivate their reputations? Asset fee levels are shown to emerge endogeneously when entry by advisers is costly. Fees cannot serve as signaling device because uninformed advisers who do not mimic the asset and performance fees offered by the informed adviser thereby reveal their type and receive no funds to manage. Portfolio choice does serve as a signaling device. This paper shows:

- Incentives to build a reputation induce distortions in risk-averse investment advisers' portfolio choices when advisers' compensation is in the form of periodic asset fees.
- These distortions may be of two types: Uninformed advisers may seek to obscure their lack of information by trading aggressively to pool with the informed adviser (pooling equilibrium). Or, the informed adviser may seek to separate himself from the uninformed adviser by trading so aggressively that the uninformed adviser finds it too risky to attempt to mimic (separating equilibrium). In either equilibrium, the reputation effect increases the variance

- of fund returns both conditioned on the private information and unconditionally since pooling and separating strategies both increase portfolio risk.
- Whether a pooling or separating equilibrium obtains depends on advisers' risk-aversion and the precision of the informed adviser's information. Higher precision and greater risk aversion both favor the separating equilibrium. When advisers are very risk averse, only the separating equilibrium is possible.
 - Asset fees emerge endogenously when free entry by agents who become uninformed advisers is allowed. Both pooling and separating equilibria continue to exist in this case. Numerical examples show that asset fees may increase or decrease in the separating equilibrium as uninformed advisers enter. In the pooling equilibrium, asset fees fall as the proportion of uninformed advisers increases.
 - For plausible parameter values, a pooling equilibrium with roughly equal numbers of informed and uninformed advisers seems likely.
 - Performance fees (a misnomer, since such fees do not create incentives to acquire costly information, but instead cause advisers to alter portfolio weightings) can redress portfolio distortions induced by advisers' reputational concerns. This observation rationalizes the otherwise puzzling role of performance fees. In the separating equilibrium, the first-best can be achieved. In the pooling equilibrium, strict improvements in welfare of the investors is possible. In either case, the welfare of the advisers are unaffected by the imposition of performance fees because the adviser always alters his equilibrium portfolio choice when a performance fee is imposed to achieve exactly the same payoff, state-by-state.

Taking asset trading costs into account, performance fees are most likely to be used when (i) there are many investors with small amounts of wealth to invest, (ii) assets are difficult to buy directly (e.g., foreign stocks), or (iii) benchmark portfolios are not tradable (e.g., real estate). The analysis also suggests these mechanisms are most likely to be used when reputation considerations are acute: when young advisers have little history of investment returns or when differences in ability across managers are small.

When a single adviser independently operates just one fund, there is a very strong link between the adviser's compensation and the performance of the fund. As mutual funds are joined together in complexes, it becomes possible to weaken the link between the compensation an individual adviser and the performance of the fund he manages. The owner of a complex may adopt a portfolio approach with respect to his stable of advisers by encouraging each of them to bet aggressively so that some of them will generate returns consistent with superior information. Basically, the complex can serve as a syndicate that absorbs some of the risk associated with making aggressive portfolio choices. Thus, the emergence of fund complexes may have the effect of making fund managers

behave in a less risk-averse fashion. This change in the mutual fund industry should have the same effect as reducing the risk aversion of the manager and thus favor the pooling equilibrium. $\square$

Appendix A. Proofs

Lemma A.1. Given the uninformed adviser seeks to mimic the informed adviser, who is playing strategy $s_I((-\hat{\omega}, \hat{\omega}); (1, 0), (0, 1))$, strategies $s_U((-\hat{\omega}, \hat{\omega}); (q, 1 - q))$ by the uninformed adviser where $q \notin [1 - p, p]$ are dominated.

Proof. Table 2 lists the first period outcomes from the strategies. In cases 1, 3, 6, or 8 (see Table 2), the uninformed adviser chooses the same weight as the informed adviser. In these cases, investors have no information to use in updating their beliefs about which adviser is informed. For any choices of p and q , the probability that one of these cases obtains is $\frac{1}{2}$. Hence, the uninformed advisers' optimal choice of q depends only on the inferences investors drawn when one of cases 2, 4, 5, or 7 obtains. In these cases, the choices of the two advisers differ and one adviser outperforms the other. Note that cases 2 and 7 are an information set from investors' perspectives. Likewise, cases 4 and 5 are an information set from investors' perspectives.

There are three situations to consider: (I) $q \in [1 - p, p]$, (II) $q < 1 - p$, and (III) $p < q$.

Situation I: $q \in [1 - p, p]$ Case 2 is at least as likely as case 7 and case 5 is at least as likely as case 4. Hence, investors' posterior beliefs are that the adviser who performed best is most likely to be informed. In this situation, the probability the uninformed adviser manages all of the investors' wealth in period 2 is: $\text{Prob}(\text{Case 4}) + \text{Prob}(\text{Case 7}) = (1 - p)(1 - q)/2 + (1 - p)q/2 = (1 - p)/2$.

In situations (II) and (III) The investors' inference as to which adviser is informed depends on the adviser's portfolio choice, to the detriment of the uninformed adviser.

Situation II: $q < 1 - p$. This implies case 4 is more likely than case 5, and case 2 is more likely than case 7. Thus, when advisers choose different portfolio weights, investors infer the adviser who chose $\hat{\omega}$ is most likely to be informed and, consequently, investors allocate their assets to that adviser in the second period. In this situation, the probability that the uninformed adviser manages all investors' assets in the second period is: $\text{Prob}(\text{Case 5}) + \text{Prob}(\text{Case 7}) = pq/2 + (1 - p)q/2 = q/2 < (1 - p)/2$.

Situation III: $p < q$. This implies case 5 is more likely than case 4, and case 7 is more likely than case 2. Thus, when advisers choose different portfolio weights, investors infer the adviser who chose $\hat{\omega}$ is most likely to be informed and, consequently, investors allocate their assets to that adviser in the second period. In this situation, the probability that the uninformed adviser manages all

investors' assets in the second period is: $\text{Prob}(\text{Case } 2) + \text{Prob}(\text{Case } 4) = p(1 - q)/2 + (1 - p)(1 - q)/2 = (1 - q)/2 < (1 - p)/2$.

In either situation II or III, the adviser is less likely to manage all investor wealth in period 2 than in situation I, hence the uninformed adviser chooses $q \in [1 - p, p]$.

Proof of Proposition 1. The proof proceeds in three parts. First, it is shown that when Eq. (4) holds, the uninformed adviser prefers pooling to the best (from his perspective) separating equilibrium. Second, it is shown that given investors believe a deviator is uninformed, both advisers prefer the pooling equilibrium to any deviation. Finally, it is shown that the investors' beliefs supporting the equilibrium are not contradicted by the Intuitive Criterion.

Given separation, the best payoff for the uninformed adviser follows from choosing $\omega = 0$ with probability 1 in each period. The payoff from this strategy is $U(f/2)$. From Table 2, the payoff to the uninformed adviser from mimicking the informed adviser at $\omega \in (0, 1]$ is

$$\begin{aligned}
 M(\omega) &= \frac{pq}{2} U[f(\tfrac{1}{2}(1 + \omega) + \tfrac{1}{2}(1 - f)(1 + \omega))] + \frac{p(1 - q)}{2} U[f(\tfrac{1}{2}(1 - \omega) + 0)] \\
 &\quad + \frac{(1 - p)q}{2} U[f(\tfrac{1}{2}(1 - \omega) + \tfrac{1}{2}(1 - f)(1 - \omega))] \\
 &\quad + \frac{(1 - p)(1 - q)}{2} U[f(\tfrac{1}{2}(1 + \omega) + (1 - f)[\tfrac{1}{2}(1 + \omega) + \tfrac{1}{2}(1 - \omega)])] \\
 &\quad + \frac{pq}{2} U[f(\tfrac{1}{2}(1 - \omega) + 0)] + \frac{p(1 - q)}{2} U[f(\tfrac{1}{2}(1 + \omega) \\
 &\quad + \tfrac{1}{2}(1 - f)(1 + \omega))] + \frac{(1 - p)q}{2} U[f(\tfrac{1}{2}(1 + \omega) + (1 - f)[\tfrac{1}{2}(1 + \omega) \\
 &\quad + \tfrac{1}{2}(1 - \omega)])] + \frac{(1 - p)(1 - q)}{2} U[f(\tfrac{1}{2}(1 - \omega) + \tfrac{1}{2}(1 - f)(1 - \omega))] \\
 &= \frac{p}{2} U\left(\frac{f(2 - f)}{2}(1 + \omega)\right) + \frac{1 - p}{2} U\left(\frac{f(2 - f)}{2}(1 - \omega)\right) \\
 &\quad + \frac{p}{2} U\left(\frac{f}{2}(1 - \omega)\right) + \frac{1 - p}{2} U\left(\frac{f}{2}((2 - f)(1 + \omega) + (1 - f)(1 - \omega))\right).
 \end{aligned}
 \tag{A.1}$$

$M(\omega)$ is the sum of concave utility functions, and so is also concave. Hence, $M(\omega) \geq \min\{M(1), M(0)\}$ for all $\omega \in (0, 1]$. By choosing ω arbitrarily small but strictly positive, the payoff from mimicry to the uninformed adviser can be made

arbitrarily close to $M(0)$, yet investors can observe differences in advisers' returns. Then, for $\alpha > 0$ the payoff from mimicry at ω is:

$$\begin{aligned} M(\omega) &\geq \min\{M(0), M(1)\} \\ &= \min\left\{\frac{p}{2}U\left(\frac{f(2-f)}{2}\right) + \frac{1-p}{2}U\left(\frac{f(2-f)}{2}\right) + \frac{p}{2}U\left(\frac{f}{2}\right) \right. \\ &\quad \left. + \frac{1-p}{2}U\left(\frac{f}{2}(3-2f)\right), \frac{p}{2}U(f(2-f)) + \frac{1-p}{2}U(f(2-f))\right\} \\ &\geq \min\left\{\frac{1}{2}U\left(\frac{f(2-f)}{2}\right) + \frac{1}{2}U\left(\frac{f}{2}\right), \frac{1}{2}U(f(2-f))\right\}. \end{aligned}$$

When $f < 1$, $\frac{1}{2}U(f(2-f)/2) + \frac{1}{2}U(f/2)$ exceeds $U(f/2)$. When inequality (4) holds, $\frac{1}{2}U(f(2-f))$ exceeds $U(f/2)$. Thus, the worst payoff from mimicry exceeds the payoff from choosing $\omega = 0$ whenever inequality (4) is satisfied. Inequality (4) holds for some positive f when $\alpha > \frac{1}{2}$. This establishes the non-existence of a separating equilibrium.

Sufficient conditions for $s_U(\Omega; \Pi)$ and $s_I(\Omega; \Pi^B, \Pi^G)$ to form a pooling equilibrium are that the respective equilibrium payoffs of the uninformed and informed advisers exceed

$$\begin{aligned} EU(f\tilde{R}_{u2}(\gamma_{u1}\tilde{R}_{u1} + \gamma_{u2}(1-f)\tilde{I})) &= U\left(\frac{f}{2}\right) \\ \text{for } \tilde{R}_{u2} &= 1, \tilde{R}_{u1} = 1, \gamma_{u1} = \frac{1}{2} \quad \text{and} \quad \gamma_{u2} = 0 \end{aligned}$$

and

$$\begin{aligned} EU(f\tilde{R}_{i2}(\gamma_{i1}\tilde{R}_{i1} + \gamma_{i2}(1-f)\tilde{I})) &\quad \text{for } \gamma_{i1} = \frac{1}{2} \quad \text{and} \quad \gamma_{i2} = 0 \\ &= [p(1+\hat{\omega})^x + (1-p)(1-\hat{\omega})^x] \left[pU\left(\frac{f}{2}(1+\hat{\omega})\right) + (1-p)U\left(\frac{f}{2}(1-\hat{\omega})\right) \right]. \end{aligned} \tag{A.2}$$

These expressions are the payoffs from the best deviations by the uninformed and informed advisers given investors conclude with certainty that the deviator is uninformed and, hence, manages no money in the second period. The first term in square brackets of Eq. (A.2) is the multiplicatively separable contribution to utility from the second period portfolio choice of the informed adviser.

Conditions under which $M(\omega) \geq U(f/2)$ for $\omega \in (0, 1]$ are established in the first part of the proof. Thus, the payoff to the uninformed adviser in the pooling equilibrium exceeds the payoff from the best deviation, given investors believe the deviator is uninformed. It remains to show that the payoff to the informed

adviser in the pooling equilibrium exceeds the payoff from the any deviation given investors believe the deviator is uninformed, namely Eq. (A.2). The payoff to the informed adviser in the pooling equilibrium is

$$\begin{aligned} & [p(1 + \hat{\omega})^z + (1 - p)(1 - \hat{\omega})^z] \left[\frac{p}{2} U \left(\frac{f}{2} (1 + \omega + (1 - f)(1 + \omega + 1 - \omega)) \right) \right. \\ & + \frac{p}{2} U \left(\frac{f}{2} (1 + \omega + \frac{1}{2}(1 - f)(1 + \omega + 1 + \omega)) \right) + \frac{1 - p}{2} U \left(\frac{f}{2} (1 - \omega) \right) \\ & \left. + \frac{1 - p}{2} U \left(\frac{f}{2} (1 - \omega + \frac{1}{2}(1 - f)(1 - \omega + 1 - \omega)) \right) \right]. \end{aligned}$$

Comparing this expression to Eq. (A.2) reduces to showing

$$\begin{aligned} & \frac{p}{2} U(1 + \omega + 2(1 - f)) + \frac{p}{2} U((2 - f)(1 + \omega)) + \frac{1 - p}{2} U(1 - \omega) \\ & + \frac{1 - p}{2} U(1 - \omega + 1 - f) > pU(1 + \hat{\omega}) + (1 - p)U(1 - \hat{\omega}). \end{aligned}$$

The left-hand side of this inequality is decreasing in f for $f < 1$. At $f = 1$ and $\omega = \hat{\omega}$, there is equality. So, the inequality is strict for $f < 1$ and $\omega = \hat{\omega}$. Since the left-hand side is a continuous function of ω , there is a neighborhood $(\hat{\omega} - \delta, \hat{\omega} + \delta)$ of $\hat{\omega}$ where the condition is satisfied.

Finally, it must be shown that the beliefs of investors in the event an adviser deviates from the equilibrium, namely that the deviator is uninformed, survive the Intuitive Criterion. For the Intuitive Criterion to restrict off-equilibrium investors' beliefs, it must be that some type, either informed or uninformed, prefers the equilibrium payoff to the best payoff (over all possible reallocation choices by investors) that could be earned by a deviation, namely allocating all funds to the deviator in the second period. However,

$$M(\omega) < \max_{\omega_d} EU(f\tilde{R}_{U2}(\gamma_{U1}\tilde{R}_{U1} + \gamma_{U2}(1 - f)\tilde{I}))$$

for $\tilde{R}_{U2} = 1$, $\tilde{R}_{U1}$ implied by $\omega_d^* = \arg \max_{\omega_d} EU(f\tilde{R}_{U2}(\gamma_{U1}\tilde{R}_{U1} + \gamma_{U2}(1 - f)\tilde{I}))$, $\gamma_{U1} = \frac{1}{2}$, and $\gamma_{U2} = 1$ follows by inspection of terms. Similarly, define $N(\omega)$ to be the equilibrium payoff to the informed adviser. By inspection of terms,

$$N(\omega) < \max_{\omega_d} EU(f\tilde{R}_{I2}(\gamma_{I1}\tilde{R}_{I1} + \gamma_{I2}(1 - f)\tilde{I}))$$

for $\tilde{R}_{I2}$ implied by $\hat{\omega}$, $\tilde{R}_{I1}$ implied by $\omega_d^* = \arg \max_{\omega_d} EU(f\tilde{R}_{I2}(\gamma_{I1}\tilde{R}_{I1} + \gamma_{I2}(1 - f)\tilde{I}))$, $\gamma_{I1} = \frac{1}{2}$, and $\gamma_{I2} = 1$. Hence, the Intuitive Criterion does not rule out

any investor beliefs, including those which support the equilibrium, namely that deviators are uninformed. $\square$

Lemma A.2. Let $V(\bar{\omega}) = U(1 + \bar{\omega} + k(1 + \omega)) + U(1 - \bar{\omega} + k(1 - \omega))$. For $\alpha \leq -1$ and $\omega \in (0, 1)$, $V(1) < U(1 + \omega) + U(1 - \omega)$.

Proof. Multiplying through by $\alpha/2$ and raising both sides to the power $1/\alpha$, it is necessary to show

$$\left(\frac{(2 + k(1 + \omega))^\alpha + (k(1 - \omega))^\alpha}{2} \right)^{1/\alpha} < \left(\frac{(1 + \omega)^\alpha + (1 - \omega)^\alpha}{2} \right)^{1/\alpha}.$$

Now,

$$\left(\frac{(2 + k(1 + \omega))^\alpha + (k(1 - \omega))^\alpha}{2} \right)^{1/\alpha} < \frac{1}{2} \left(\frac{(5 + \omega)^\alpha + (1 - \omega)^\alpha}{2} \right)^{1/\alpha}$$

because the expression increases in k and $k \in (0, \frac{1}{2})$. Theorem 16 of Hardy et al. (1934) implies the right-hand side of the last expression increases in α for $\alpha \leq -1$ so the right-hand side is less than or equal to

$$\frac{(5 + \omega)(1 - \omega)}{6}$$

$$< (1 - \omega) \quad \text{since } \omega < 1$$

$$< ((1 + \omega)^\alpha + (1 - \omega)^\alpha)^{1/\alpha} \quad \text{by Theorem 4 of Hardy et al. (1934).} \quad \square$$

Lemma A.3. Let $K(\omega)$ be the Expected utility of the uninformed adviser in the event both advisers play ω in period 1. Let $L(\bar{\omega}, \omega)$ be the expected utility of the uninformed adviser in the event (i) the uninformed adviser chooses $\bar{\omega}$, (ii) the informed adviser chooses ω , and (iii) with probability one the uninformed adviser captures all of the investors' funds in period 2. For $\alpha \leq -1$, there exists a $\bar{\omega}$ such that $K(\omega) = L(\bar{\omega}, \omega)$ where $|\omega| < |\bar{\omega}| < 1$.

Proof. Take $\omega > 0$. The argument for $\omega < 0$ is symmetric.

$$K(\omega) = EU(f\tilde{R}_{u2}(\gamma_{u1}\tilde{R}_{u1} + \gamma_{u2}(1 - f)\tilde{I})) = EU\left(\frac{f}{2}(2 - f)\tilde{R}_{u1}\right)$$

since $\tilde{R}_{u2} = 1$, $\tilde{R}_{u1} = \tilde{I}$, and $\gamma_{u1} = \gamma_{u2} = \frac{1}{2}$ when both advisers choose ω ,

$$= \frac{1}{2}U\left(\frac{f}{2}(2 - f)(1 + \omega)\right) + \frac{1}{2}U\left(\frac{f}{2}(2 - f)(1 - \omega)\right). \quad (\text{A.3})$$

$$\begin{aligned}
L(\bar{\omega}, \omega) &= EU(f\tilde{R}_{u2}(\gamma_{u1}\tilde{R}_{u1} + \gamma_{u2}(1-f)\tilde{I})) \\
&= EU(f(\tfrac{1}{2}\tilde{R}_{u1} + (1-f)\tilde{I})) = \tfrac{1}{2}U(f(\tfrac{1}{2}(1-\bar{\omega}) + (1-f)(\tfrac{1}{2}(1-\bar{\omega}) \\
&\quad + \tfrac{1}{2}(1-\omega)))) + \tfrac{1}{2}U(f(\tfrac{1}{2}(1+\bar{\omega}) + (1-f)(\tfrac{1}{2}(1+\bar{\omega}) + \tfrac{1}{2}(1+\omega))))).
\end{aligned}
\tag{A.4}$$

since $\tilde{R}_{u2} = 1$, $\gamma_{u1} = \frac{1}{2}$, and $\gamma_{u2} = 1$ by (iii). Comparing Eqs. (A.3) and (A.4), it is clear that $K(\omega) = L(\bar{\omega}, \omega)$ if and only if

$$U(1+\omega) + U(1-\omega) = U(1+\bar{\omega} + k(1+\omega)) + U(1-\bar{\omega} + k(1-\omega)), \tag{A.5}$$

where $k = (1-f)/(2-f)$. Let $V(\bar{\omega}) = U(1+\bar{\omega} + k(1+\omega)) + U(1-\bar{\omega} + k(1-\omega))$. Since V is continuous, Eq. (A.5) holds for some $\bar{\omega} \in (\omega, 1)$ if

$$V(1) < U(1+\omega) + U(1-\omega) < V(\omega).$$

The left inequality follows from Lemma 1. The right inequality is immediate because $k > 0$. $\square$

Lemma A.4. If $\alpha \leq -1$ then there is no sequential equilibrium that satisfies the Intuitive Criterion in which the informed and uninformed advisers both choose portfolio ω with positive probability.

Proof. Suppose both advisers choose ω with positive probability after the informed adviser receives signal G . By Lemma A.3, there exists $\bar{\omega} \in (\omega, 1)$ such that the uninformed adviser prefers (i) choosing ω in period 1 and capturing half of the investors' funds in period 2 to (ii) choosing $\bar{\omega}$ in period 1 and capturing all of the investors' funds in period 2. Thus, there do not exist out-of-equilibrium beliefs for the investors that rationalize a choice of $\bar{\omega}$ by the uninformed adviser. If $\bar{\omega}$ is observed, the investors must conclude that the adviser who selected $\bar{\omega}$ is informed. Hence, in period 2, the investors allocate all their wealth to the adviser who selected $\bar{\omega}$ (i.e., the informed adviser) in period 1.

It remains to show that the informed adviser prefers $\bar{\omega}$ to ω , given that selecting $\bar{\omega}$ results in investors allocating all their wealth to the informed adviser.

The expected utility to the informed adviser from $\bar{\omega}$ is

$$EU(f\tilde{R}_{t2}(\gamma_{t1}\tilde{R}_{t1} + \gamma_{t2}(1-f)\tilde{I})) = EU(f\tilde{R}_{t2}(\tfrac{1}{2}\tilde{R}_{t1} + (1-f)\tilde{I}))$$

since $\gamma_{t1} = \frac{1}{2}$ and (by the argument above) $\gamma_{t2} = 1$,

$$\begin{aligned}
&= [p(1+\hat{\omega})^2 + (1-p)(1-\hat{\omega})^2][pU(f(\tfrac{1}{2}(1+\bar{\omega}) + (1-f)[\tfrac{1}{2}(1+\bar{\omega}) \\
&\quad + \tfrac{1}{2}(1+\omega)])) + (1-p)U(f(\tfrac{1}{2}(1-\bar{\omega}) + (1-f)[\tfrac{1}{2}(1-\bar{\omega}) + \tfrac{1}{2}(1-\omega)]))].
\end{aligned}
\tag{A.6}$$

The expected utility to the informed adviser from choosing ω in period 1 is

$$EU(f\tilde{R}_{t2}(\gamma_{t1}\tilde{R}_{t1} + \gamma_{t2}(1-f)\tilde{I})) = EU(f\tilde{R}_{t2}(\frac{1}{2}\tilde{R}_{t1} + \frac{1}{2}(1-f)\tilde{I}))$$

since $\gamma_{t1} = \gamma_{t2} = \frac{1}{2}$,

$$= [p(1 + \hat{\omega})^\alpha + (1-p)(1 - \hat{\omega})^\alpha][pU(f(\frac{1}{2}(1 + \omega) + \frac{1}{2}(1-f)(1 + \omega))) + (1-p)U(f(\frac{1}{2}(1 - \omega) + \frac{1}{2}(1-f) + (1 - \omega)))]. \quad (\text{A.7})$$

It must be shown that Eq. (A.7) < Eq. (A.6). This reduces to showing

$$pU(1 + \omega) + (1-p)U(1 - \omega) < pU(1 + \bar{\omega} + k(1 + \omega)) + (1-p)U(1 - \bar{\omega} + k(1 - \omega))$$

where $k = (1-f)/(2-f)$. Subtracting $(1-p)$ times (A.5) from this gives

$$(2p-1)U(1 + \omega) < (2p-1)U(1 + \bar{\omega} + k(1 + \omega)).$$

This reduces to $\omega - k(1 - \omega) < \bar{\omega}$, which holds for all $\omega \in (0, 1)$ and $f \in (0, 1)$.

Therefore, the informed adviser prefers $\bar{\omega}$ to ω and the investors must conclude the selection of $\bar{\omega}$ could only be made by the informed adviser. This breaks the equilibrium. By symmetry, an identical argument applies if the informed adviser receives signal B in period 1.

Proof of Proposition 2. From Remark 1, the uninformed adviser will only play $\omega \neq 0$ with positive probability if the informed adviser plays ω with positive probability. From Lemma A.4, this cannot happen; so the uninformed adviser must play $\omega = 0$ with probability 1 in equilibrium. The informed adviser will choose a first-period portfolio equal to the single-period optimum ($\hat{\omega}$ following signal G , $\check{\omega}$ following signal B), provided the uninformed adviser does not find it worthwhile to mimic this strategy. From the proof of Proposition 1, the payoff to the uninformed adviser from mimicking the informed adviser at ω is Eq. (A.1). The payoff from choosing $\omega = 0$ with probability 1 is $U(f/2)$. Therefore, to discourage mimicking, the informed adviser must choose $\hat{\omega}$ and $\check{\omega}$ far enough away from 0 so that $U(f/2) = M(\omega)$. Simplifying this expressions yields Eq. (5).

Nothing essential to the proof requires that the number of strategies played by either of the advisers with positive probability be finite. If there were a countable number of strategies played with positive probability, the argument applies by redefining Ω and Π to be sequences. If a continuum of strategies is played, the argument proceeds by considering the supports of the distributions over strategies. $\square$

Proof of Proposition 3. The variance in portfolio returns given portfolio choice ω with respect to the probability distribution of asset returns conditioned on the

signal is

$$\text{Var}_p(\omega\tilde{A} + (1 - \omega)M) = 4p(1 - p)\omega^2.$$

The unconditioned variance has $p = \frac{1}{2}$. Both the conditioned and unconditioned variances increases as ω increases in absolute value. Since the reputation effect leads to a higher (in absolute value) choice of ω for one adviser in either the pooling equilibrium or the separating equilibrium, the variance of a mutual fund returns is greater in the presence of reputation effects.

Lemma A.5. Extend the definition of the utility function to negative arguments so that $U(W) = -\infty$ for $W < 0$, and assume that if an adviser does not offer a contract to investors, he receives some finite reservation utility, $\bar{U} \geq U(0)$. Then no risk-averse adviser will offer a contract where $f \leq 0$.

Proof. From Eq. (12), for asset fee $f \leq 0$ and performance fee g , the one-period payment to an adviser who chooses portfolio weight ω is

$$\begin{aligned} f(\omega\tilde{A} + (1 - \omega)M) + g(\omega\tilde{A} + (1 - \omega)M - \lambda\tilde{A} - (1 - \lambda)M) \\ = (f\omega + g(\omega - \lambda))\tilde{A} + (f(1 - \omega) + g(\lambda - \omega))M \leq g(\omega - \lambda)\tilde{A} \\ + g(\lambda - \omega)M \quad \text{for } f \leq 0. \end{aligned}$$

Thus, for any $f \leq 0$ the expected utility of the adviser is less than or equal to

$$\begin{aligned} pU(2g(\omega - \lambda) + g(\lambda - \omega)) + (1 - p)U(g(\lambda - \omega)) \\ = pU(g(\omega - \lambda)) + (1 - p)U(g(\lambda - \omega)). \end{aligned}$$

Now for any g , ω , and λ , the argument of one of utility functions in the last expression above must be negative, or both must be zero. The result follows since $U(W) \leq \bar{U}$ for any $W \leq 0$. $\square$

Proof of Proposition 5. Suppose the uninformed adviser plays $s_U(0; 1)$, while the informed adviser plays $s_I((\hat{\omega}, \check{\omega}); (1, 0), (0, 1))$. Choose M as the benchmark portfolio. The uninformed adviser's utility from strategy $s_U(0; 1)$ does not change when a performance fee is imposed. The informed adviser's expected utility is lower with strategy $s_I((\hat{\omega}, \check{\omega}); (1, 0), (0, 1))$ when $g \neq 0$ than when $g = 0$. However, the informed adviser can receive the same utility as when $g = 0$ by using strategy $s_I([\frac{f}{f + g}]\hat{\omega}, [\frac{f}{f + g}]\check{\omega}); (1, 0), (0, 1))$. The uninformed adviser will not find it worthwhile to mimic the informed adviser, even though the informed adviser's portfolio choice is less aggressive, because the expected utility from strategy $s_I([\frac{f}{f + g}]\hat{\omega}, [\frac{f}{f + g}]\check{\omega}); (\frac{1}{2}, \frac{1}{2}))$ is the same for all g by Proposition 4. Thus, the separating equilibrium is preserved when the performance fee is imposed. The uninformed adviser continues to choose the portfolio that, given his information, the investors prefer. The expected utilities of the advisers are

unchanged, but the portfolio choice of the informed adviser will vary with g . It remains to show that g can be chosen to optimize the expected utility of investors who invest with the informed adviser.

Recall the signals received by the informed adviser are G and B . With the performance fee, investors who invest with the informed adviser in the first period receive

$$\begin{aligned} \frac{1}{2}E\left\{U\left[\left(1-f\right)\left(\frac{f}{f+g}\hat{\omega}\tilde{A}+\left(1-\frac{f}{f+g}\hat{\omega}\right)M\right)-g\left(\frac{f}{f+g}\hat{\omega}\tilde{A}\right.\right.\right. \\ \left.\left.\left.+\left(1-\frac{f}{f+g}\hat{\omega}\right)M-1\right)\right]|G\right\}+\frac{1}{2}E\left\{U\left[\left(1-f\right)\left(\frac{f}{f+g}\check{\omega}\tilde{A}\right.\right.\right. \\ \left.\left.\left.+\left(1-\frac{f}{f+g}\check{\omega}\right)M\right)-g\left(\frac{f}{f+g}\check{\omega}\tilde{A}+\left(1-\frac{f}{f+g}\check{\omega}\right)M-1\right)\right]|B\right\}. \end{aligned}$$

Since $\hat{\omega} = -\check{\omega}$; and $\Pr(\tilde{A} = 2|G) = \Pr(\tilde{A} = 0|B) = p$, this expression reduces to

$$= pU\left[(1-f)(1+\hat{\omega})-\frac{g}{f+g}\hat{\omega}\right] + (1-p)U\left[(1-f)(1-\hat{\omega})-\frac{g}{f+g}\hat{\omega}\right].$$

Choosing g to satisfy

$$(1-f)(1+\hat{\omega})-\frac{g}{f+g}\hat{\omega} = (1-f)(1+\hat{\omega})$$

provides the investors with the one-period utility they would obtain in the absence of reputation concerns. This is a strict improvement whenever $\hat{\omega} < \hat{\omega}$. $\square$

Proof of Proposition 6. Suppose the uninformed adviser plays $s_U((\omega, -\omega); (q, 1-q))$, while the informed adviser plays $s_I((\omega, -\omega); (1, 0), (0, 1))$. Choose M as the benchmark portfolio. The ex ante first-period utility of an investor in the absence of a performance fee is

$$\begin{aligned} \frac{1}{2}pU((1-f)(1+\omega)) + (1-p)U((1-f)(1-\omega)) \\ + \frac{1}{2}U((1-f)(1+\omega)) + \frac{1}{2}U((1-f)(1-\omega)) \\ = \frac{1+2p}{4}U((1-f)(1+\omega)) + \frac{3-2p}{4}U((1-f)(1-\omega)). \end{aligned} \quad (\text{A.8})$$

Expression (A.8) has the same form as Eq. (1). The portfolio weight depends on the precision of information about the payoff on the risky asset $\tilde{A}$. Since the decisions of the informed and uninformed advisers are mixed together from the perspective of the investor, the optimal portfolio weight ω^* is calculated using

$q = (1 + 2p)/4$ in Eq. (2). Similar to Proposition 5, choosing g to satisfy

$$(1 - f)(1 + \omega) - \frac{g}{f + g}\omega = (1 - f)(1 + \omega^*)$$

improves the expected utility of investors. $\square$

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